Graphics & Visualization

Chapter 2

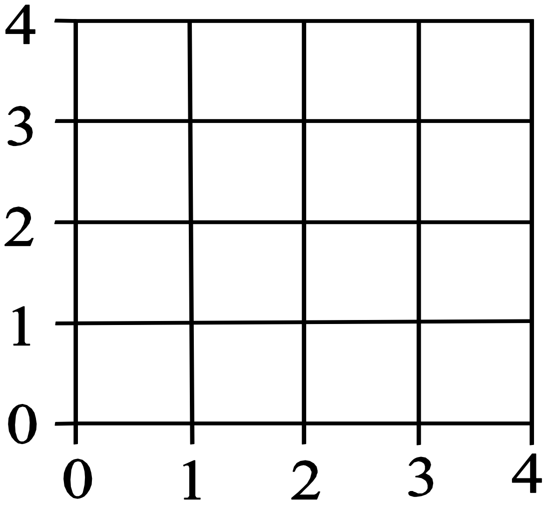
Rasterization Algorithms

Rasterization

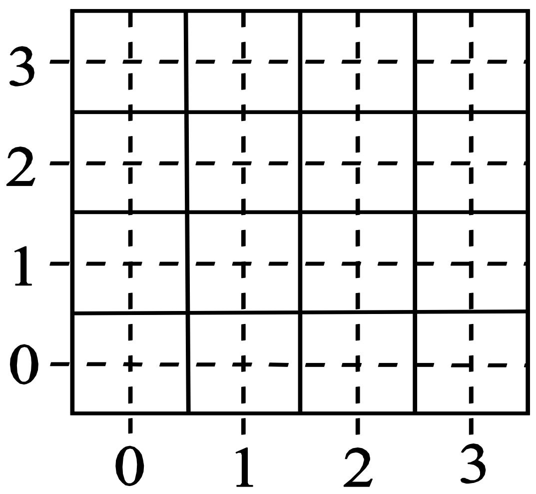
- 2D display devices consist of discrete grid of pixels
- **Rasterization:** converting 2D primitives into a discrete pixel representation
- The complexity of rasterization is O(Pp), where P is the number of primitives and p is the number of pixels
- There are 2 main ways of viewing the grid of pixels:
  - Half – Integer Centers
  - Integer Centers (shall be used)
- **Connectedness:** which are the neighbors of a pixel?
  - 4 – connectedness
  - 8 – connectedness
- Challenges in designing a rasterization algorithm:
  - Determine the pixels that accuracy describe the primitive
  - Efficiency

Rasterization (2)

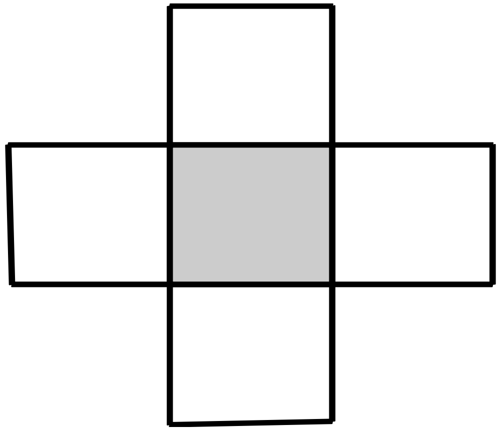
- Half – Integer Centers



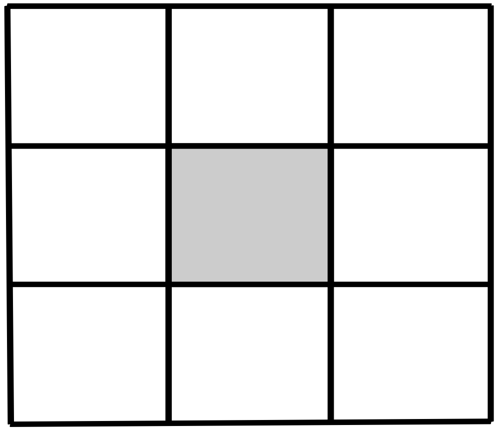
Integer Centers



- 4 – Connectedness



8 - Connectedness



Mathematical Curves

- Two mathematical forms:
  - **Implicit Form:**

e.g.:

$$f(x,y) \begin{cases} < 0, & \text{implies point(x,y) is 'inside' the curve} \\ = 0, & \text{implies point(x,y) is on the curve} \\ > 0, & \text{implies point(x,y) is 'outside' the curve} \end{cases}$$

  - **Parametric Form:**
    - Function of a parameter **t** $\in$ [0, 1]
    - **t** corresponds to arc length along the curve
    - The curve is traced as **t** goes from 0 to 1

e.g.: l(t) = (x(t), y(t))

Mathematical Curves (2)

- Examples:
  - **Implicit Form:**
    - **line:** $l(x, y) \equiv ax + by + c = 0$

      where a, b, c : line coefficients

      if l(x, y) = 0 then point (x, y) is on the curve

      else if l(x, y) < 0 then point (x, y) is on one half-plane

      else if l(x, y) > 0 then point (x, y) is on the other half-plane
    - **circle:** $c(x, y) \equiv (x - x_c)^2 + (y - y_c)^2 - r^2 = 0$

      where (x_c, y_c) : the center of the circle & r: circle's radius

      if c(x, y) = 0 then point (x, y) is on the circle

      else if c(x, y) < 0 then point (x, y) is inside the circle

      else if c(x, y) > 0 then point (x, y) is outside the circle

Mathematical Curves (3)

- Examples:
  - **Parametric Form:**
    - **line:** $\mathbf{l}(t) = (x(t), y(t))$

      where $x(t) = x_1 + t\,(x_2 - x_1)$,

      $y(t) = y_1 + t\,(y_2 - y_1)$,

      $t \in [0,1]$
    - **circle:** $\mathbf{c}(t) = (x(t), y(t))$

      where $x(t) = x_c + r\ \cos(2\pi t)$,

      $y(t) = y_c + r\ \sin(2\pi t)$,

      $t \in [0,1]$

Finite Differences

- Functions that define primitives need to be evaluated on the pixel grid for each pixel ⇨ wasteful
- Cut this cost by taking advantage of finite differences
- Forward differences (fd):
  - First (fd) : $\delta f_i = f_{i+1} - f_i$
  - Second (fd): $\delta^2 f_i = \delta f_{i+1} - \delta f_i$
  - k^{th} (fd): $\delta^k f_i = \delta^{k-1} f_{i+1} - \delta^{k-1} f_i$
- Implicit functions can be used to decide if the pixel belongs to the primitive

  e.g.: pixel(x, y) is included if $|f(x, y)| < e$,

  where e: related to the line width

Finite Differences (2)

- Examples:
  - Evaluation of the **line** function incrementally:

→from pixel (x, y) to pixel (x+1, y)

Calculation of the forward differences of the implicit line equation in the x direction from pixel x to pixel x+1:

$$\delta_x l(x,y) = l(x+1,y) - l(x,y) = a$$

Compute $l(x,y) + \delta_x l(x,y) = l(x,y) + a$

→from pixel (x, y) to pixel (x+1, y)

Calculation of the forward differences of the implicit line equation in the y direction from pixel y to pixel y+1:

$$\delta_y l(x,y) = l(x,y+1) - l(x,y) = b$$

Compute $l(x,y) + \delta_y l(x,y) = l(x,y) + b$

Finite Differences (3)

- Examples:
  - Evaluation of the **circle** function incrementally:

    →from pixel (x, y) to pixel (x+1, y)

    Calculation of the forward differences of the implicit circle equation.

    Since it has degree 2 there are two forward differences in the x direction from pixel x to pixel x+1:

$$\delta_x c(x,y) = c(x+1,y) - c(x,y) = 2(x - x_c) + 1$$

$$\delta_x^2 c(x,y) = \delta_x c(x+1,y) - \delta_x c(x,y) = 2$$

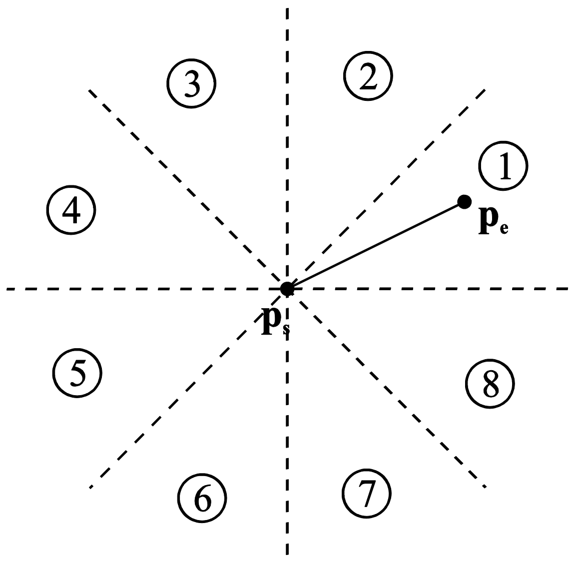
Compute $\delta_x c(x,y) = \delta_x c(x-1,y) + \delta_x^2 c(x,y)$

$$c(x+1,y) = c(x,y) + \delta_x c(x,y)$$

→from pixel (x, y) to pixel (x, y+1): similar by adding $\delta_y c(x,y)$ and $\delta_y^2 c(x,y)$

Line Rasterization

- Desired qualities of a line rasterization algorithm:
  - Selection of the nearest pixels to the mathematical path of the line
  - Constant line width, independent of the slope of the line
  - No gaps
  - High efficiency



The 8 octants with an example line in the first octant

Line Rasterization Algorithm 1

- Draw a line from pixel $p_s = (x_s, y_s)$ to pixel $p_e = (x_e, y_e)$ in the first octant

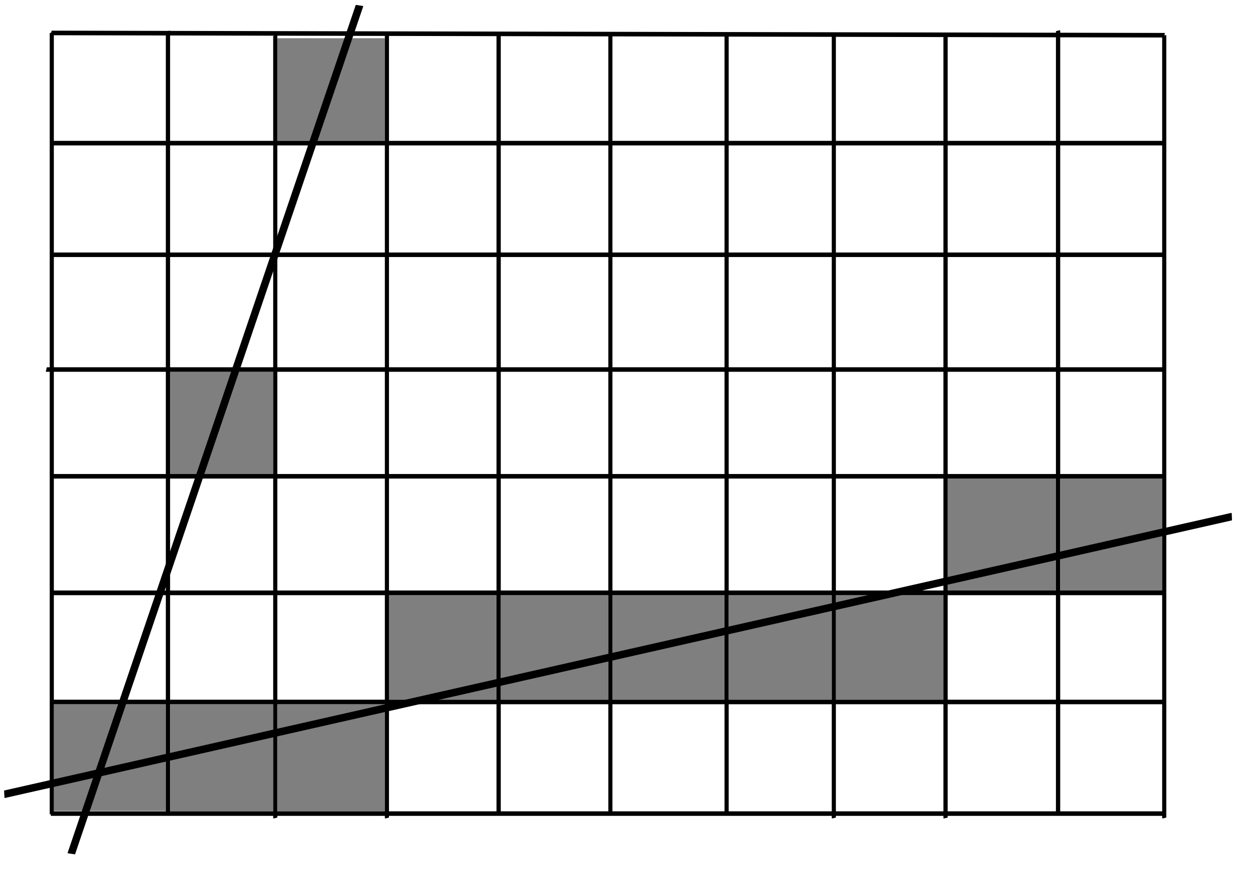
- Slope of the line: $s = \dfrac{y_e - y_s}{x_e - x_s}, \quad y = y_s + round(s \cdot (x - x_s)), \quad x = x_s, \ldots, x_e$

Algorithm:

```
line1 ( int xs, int ys, int xe, int ye, colour c )  {
        float s; int x, y;
        s = (ye - ys) / (xe - xs);  (x, y) = (xs, ys);
        while (x <= xe) {
                setpixel (x, y, c);
                x = x + 1;
                y = ys + round(s * (x - xs));
        }
}
```

Line Rasterization Algorithm 1 (2)

- Using `line1` algorithm in the first and second octants:



Line Rasterization Algorithm 2

- Avoid rounding operation by splitting y value into an integer and a float part e
- Compute its value incrementally

Algorithm:

```
line2 ( int xs, int ys, int xe, int ye, colour c )  {
    float s, e;  int x, y;
    e = 0;    s = (ye - ys) / (xe - xs);    (x, y) = (xs, ys);
    while (x <= xe) {
        /* assert -1/2 <= e < 1/2 */
        setpixel(x, y, c);
        x = x + 1;
        e = e + s;
        if (e >= 1/2) {
            y = y + 1;
            e = e - 1;
        }
    }
}
```

Line Rasterization Algorithm 2 (2)

- Algorithm `line2` resembles the leap year calculation

  - The slope is added to the *e* variable at each iteration until it makes up more than half a unit & then the line leaps up by 1.
  - The integer *y* variable is incremented and *e* is correspondingly reduced, so that the sum of the 2 variables is unchanged.

  - Similarly, the year has approximately 365,25 days but calendars are designed with an integer number of days.
  - We add a day every 4 years to make up for the error being accumulated.

Bresenham Line Algorithm

- Replace the floating point variables in `line2` by integers
- Multiplying the leap decision variables by $dx = x_e - x_s$ makes s and e integers
- The leap decision becomes $e \geq \left\lfloor \frac{dx}{2} \right\rfloor$ because e is integer
- $\left\lfloor \frac{dx}{2} \right\rfloor$ can be computed by a numerical shift
- For more efficiency replace the test $e \geq \left\lfloor \frac{dx}{2} \right\rfloor$ by $e \geq 0$ using an initial subtraction of $\left\lfloor \frac{dx}{2} \right\rfloor$ from e

Bresenham Line Algorithm (2)

- Floating point variables are replaced by integers

Algorithm

```
line3 ( int xs, int ys, int xe, int ye, colour c )  {
    int x, y, e, dx, dy;
    e = - (dx >> 1); dx = (xe - xs); dy=(ye - ys); (x, y)=(xs, ys);
    while (x <= xe) {
        /* assert -dx <= e < 0 */
        setpixel(x, y, c);
        x = x + 1;
        e = e + dy;
        if (e >= 0) {
            y = y + 1;
            e = e - dx;
        }
    }
}
```

Bresenham Line Algorithm (3)

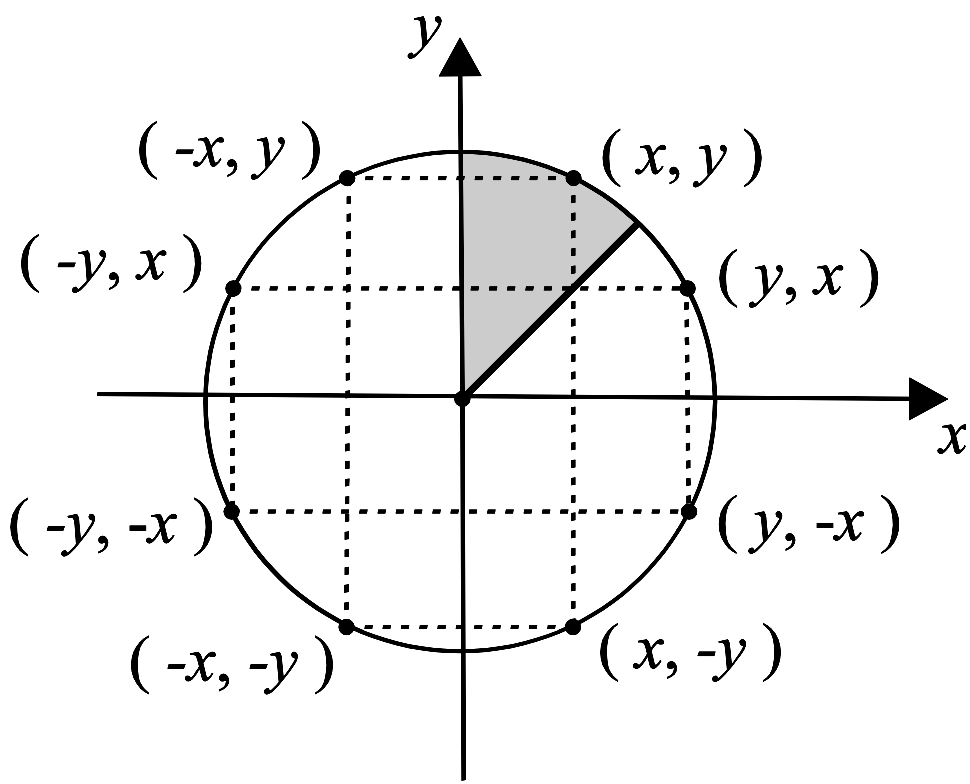
- Suitable for lines in the first octant
- Changes for other octants according to the following table

Octant	Major Axis	Minor Axis Variable
1	x	increasing
2	y	increasing
3	y	decreasing
4	x	increasing
5	x	decreasing
6	y	decreasing
7	y	increasing
8	x	decreasing

- Meets the requirements of a good line rasterization algorithm

Circle Rasterization

- Circles possess 8–way symmetry
- Compute the pixels of one octant
- Pixels of other octants are derived using the symmetry



Circle Rasterization Algorithm

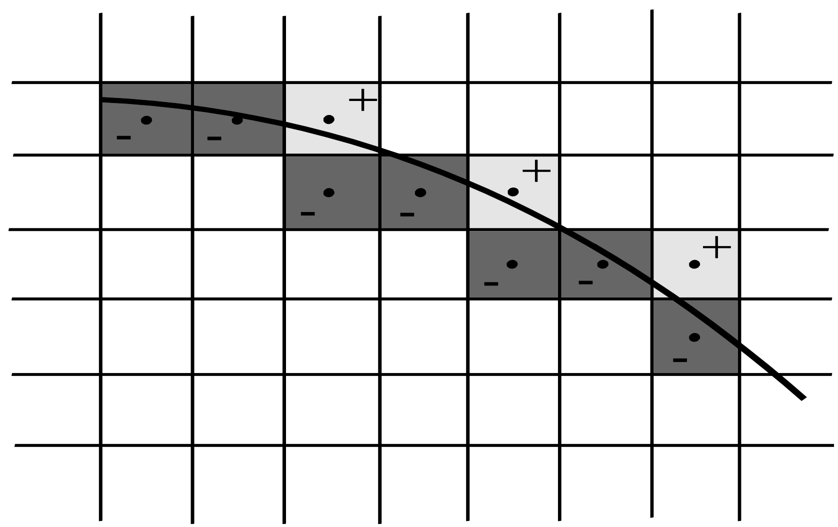
- The following algorithm exploits 8–way symmetry

Algorithm:

```
set8pixels ( int x, y, colour c )  {
    setpixel(x, y, c);
    setpixel(y, x, c);
    setpixel(y, -x, c);
    setpixel(x, -y, c);
    setpixel(-x, -y, c);
    setpixel(-y, -x, c);
    setpixel(-y, x, c);
    setpixel(-x, y, c);
}
```

Bresenham Circle Algorithm

- The radius of the circle is r
- The center of the circle is pixel (0, 1)
- The algorithm starts with pixel (0, r)
- It draws a circular arc in the second octant
- Coordinate x is incremented at every step
- If the value of the circle function becomes non-negative (pixel not inside the circle), y is decremented



Bresenham Circle Algorithm (2)

- To center the selected pixels on the circle use a circle function which is displaced by half a pixel upwards; the circle center becomes (0, ½)

$$c(x,y) = x^2 + (y - \frac{1}{2})^2 - r^2 = 0$$

- Initialize the error variable to:

$$c(0,r) = (r - \frac{1}{2})^2 - r^2 = \frac{1}{4} - r$$

- Since error is an integer variable the ¼ can be dropped
- *e* keeps the value of the implicit circle function
- For the incremental evaluation of *e* use the finite differences of that function for the 2 possible steps of the algorithm

$$c(x+1,y) - c(x,y) = (x+1)^2 - x^2 = 2x+1$$

$$c(x,y-1) - c(x,y) = (y - \frac{3}{2})^2 - (y - \frac{1}{2})^2 = -2y+2$$

Bresenham Circle Algorithm (3)

Algorithm:

```
circle ( int r, colour c )  {
    int x, y, e;
    x = 0;    y = r;    e = - r;
    while (x <= y) {
        /* assert e == x^2 + (y - 1/2)^2 - r^2 */
        set8pixels(x, y, c);
        e = e + 2 * x + 1;
        x = x + 1;
        if (e >= 0) {
            e = e  -  2 * y + 2;
            y = y - 1;
         }
    }
}
```

Point in Polygon Tests

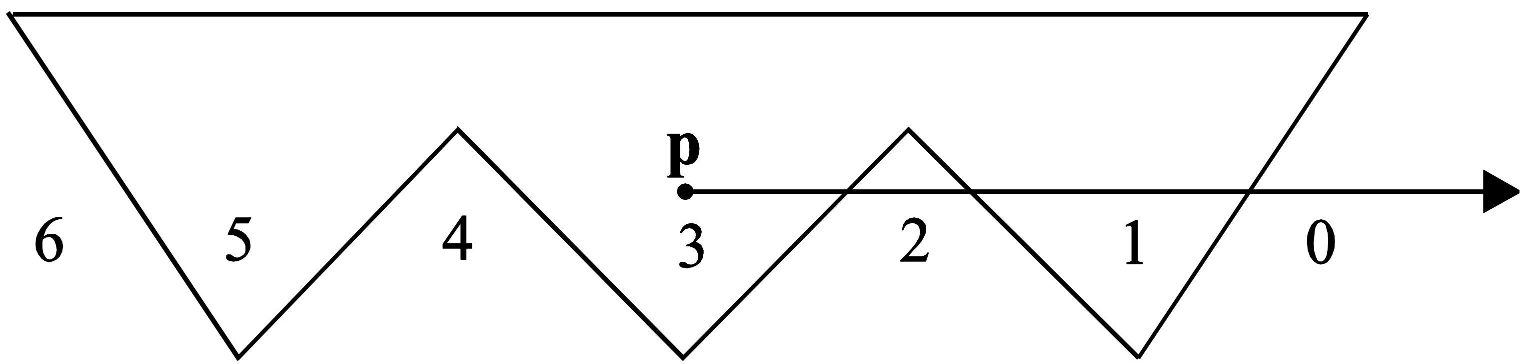
- Polygon: $\left.\begin{array}{l}\text{n vertices } (v_0, \ldots, v_{n-1}) \\ \text{n edges}\end{array}\right\}$ $\left.\right\}$ form a closed curve $v_0, v_1, \ldots, v_{n-1}, v_0$

- **Jordan Curve Theorem:** A continuous simple closed curve in the plane separates the plane into 2 regions. The ‘inside’ and the ‘outside’

- For efficient rasterization we need to know if a pixel p is inside a polygon P. There are two types of inclusion tests:
  - Parity test
  - Winding number

Point in Polygon Tests (2)

- Parity Test:
  - Draw a half line from pixel p in any direction
  - Count the number of intersections of the line with the polygon P
  - If #intersections == odd number then p is inside P
  - Otherwise p is outside P

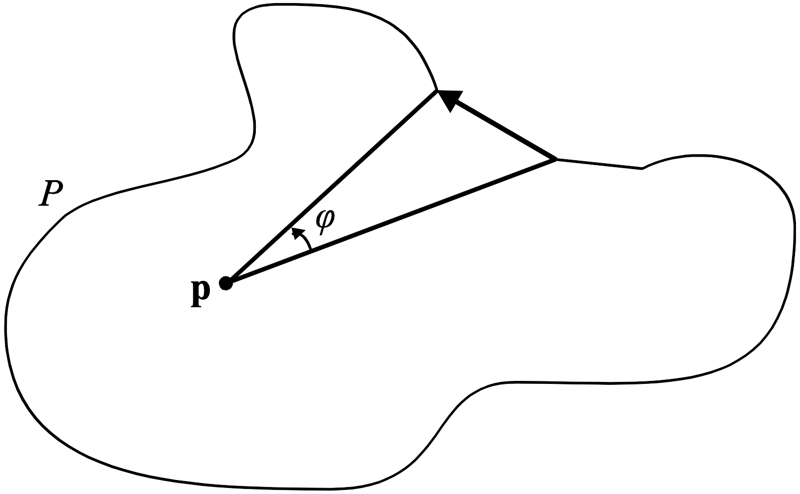


Point in Polygon Tests (3)

- Winding Number Test:
  - $\omega(P, p)$ counts the # of revolutions completed by a ray from p that traces P

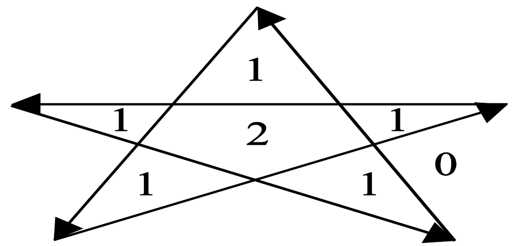
$$\omega(P,p) = \frac{1}{2\pi}\int d\varphi$$

  - For every counterclockwise revolution $\omega(P, p)$ ++
  - For every clockwise revolution $\omega(P, p)$--
  - If $\omega(P, p)$ is odd then p is inside P
  - Otherwise p is outside P

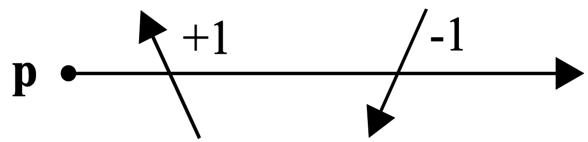


Point in Polygon Tests (4)

- The winding number test for point in polygon:

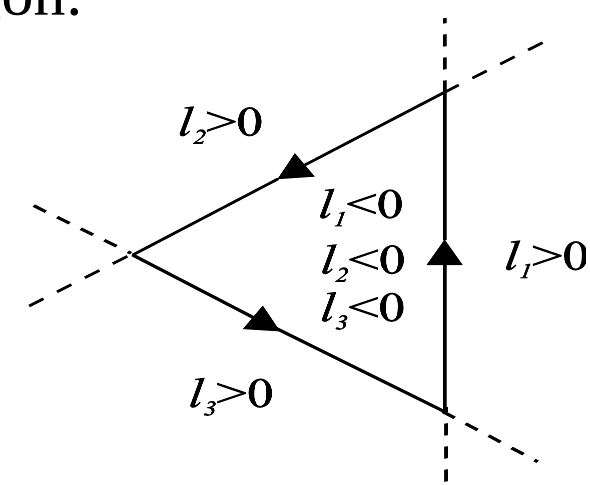


- Simple computation of the winding number:



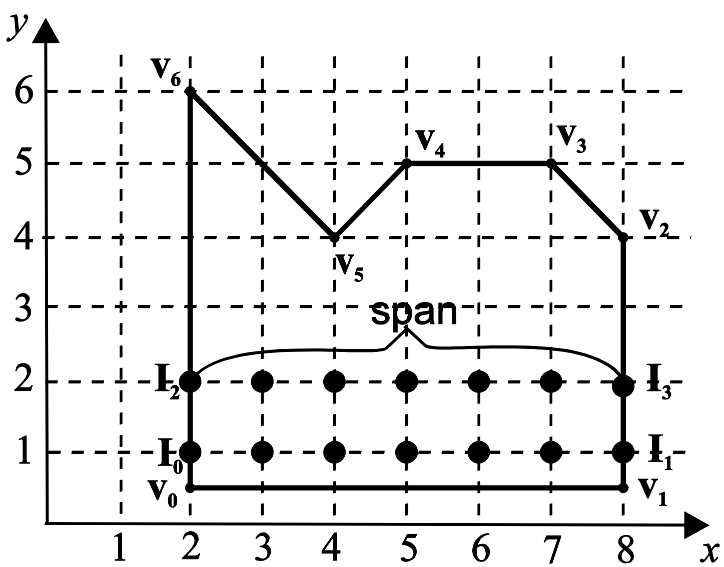
- The sign test for point in convex polygon:

$$\mathrm{sign}(l_o(\mathbf{p})) = \mathrm{sign}(l_1(\mathbf{p})) = \ldots = \mathrm{sign}(l_{n-1}(\mathbf{p}))$$



Polygon Rasterization

- Basic Polygon Rasterization Algorithm:
  - Based on the parity test
  - Steps:
    1. Compute intersections I(x, y) of every edge with all the scanlines it intersects & store them in a list
    2. Sort the intersections by (y, x)
    3. Extract spans from the list & set the pixels between them



Singularities

- Basic Polygon Rasterization Algorithm:
  - inefficient due to the cost of intersection computations

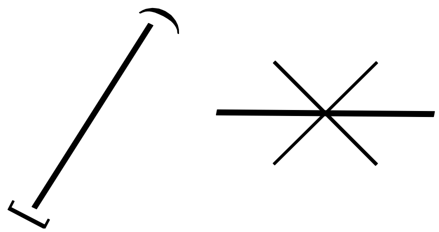
- Problem:
  - if a polygon vertex falls exactly on a scanline:

    count 2, 1 or 0 intersections ?

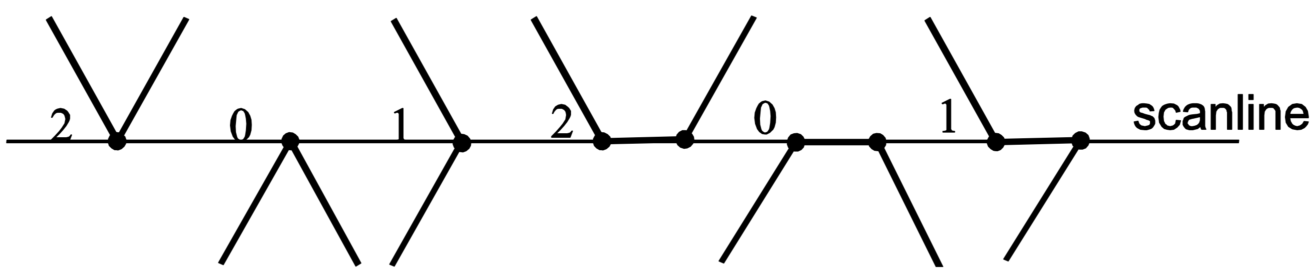
- Solutions:
  - regard edge as closed on the vertex with min y and open on the vertex with max y
  - ignore horizontal edges

Singularities (2)

- Rule for Treating Intersection Singularities



- Effect of Singularities Rule on Singularities



Scanline Polygon Rasterization Algorithm

- Takes advantage of scanline coherence & edge coherence
- Uses an Edge Table (ET) and an Active Edge Table (AET)

Algorithm:

1. Construct the polygon ET, containing the maximum y, the min x and the inverse slope of each edge (y_{max}, x_{min}, $1/s$). The record of an edge is inserted in the bucket of its minimum y coordinate.
2. For every scanline y that intersects the polygon in an upward sweep
   (a) Update the AET edge intersections for the current scanline:

   $x = x + 1/s$.

   (b) Insert edges from y bucket of ET into AET.

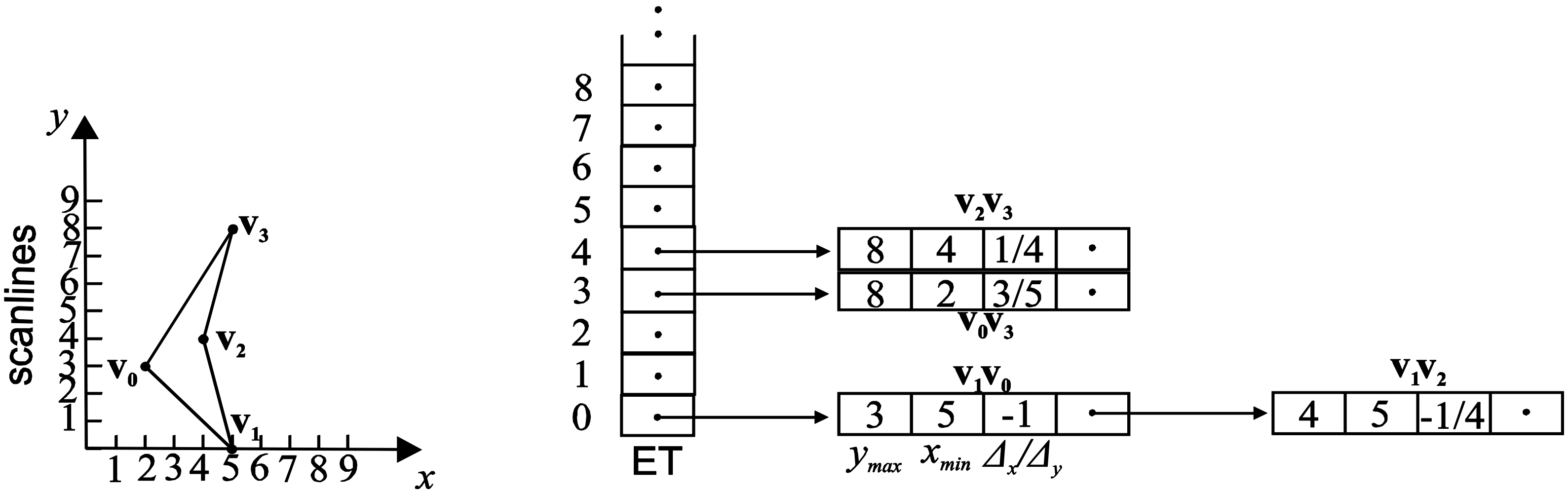
   (c) Remove edges from AET whose $y_{max} \leqslant y$.

   (d) Re-sort AET on x.

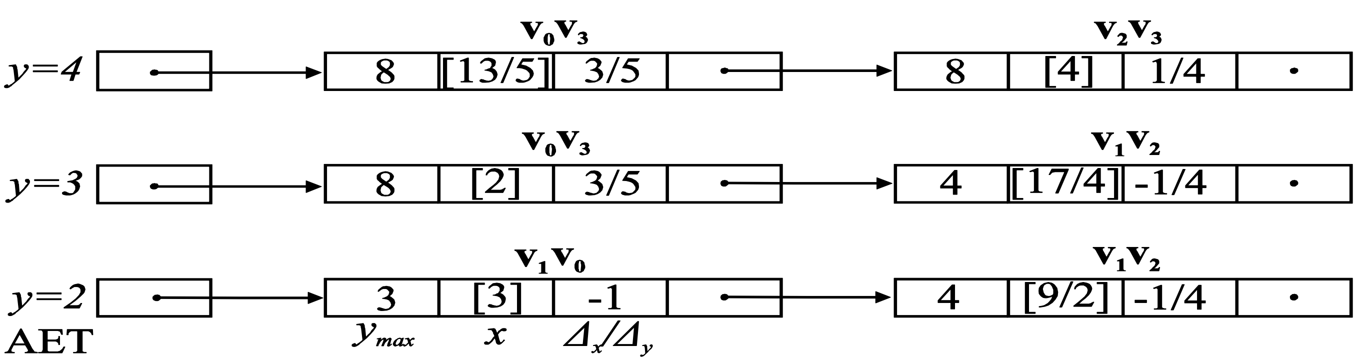
   (e) Extract spans from the AET and set their pixels.

Scanline Polygon Rasterization Algorithm (2)

- A polygon and its Edge Table (ET)

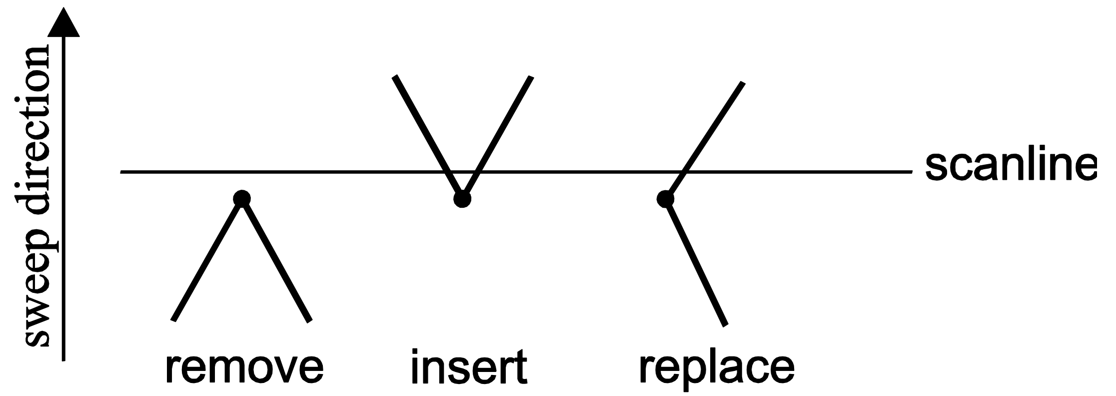


- Example states of the AET



Scanline Polygon Rasterization Algorithm (3)

- The edges that populate the AET change at polygon vertices according to the following figure:

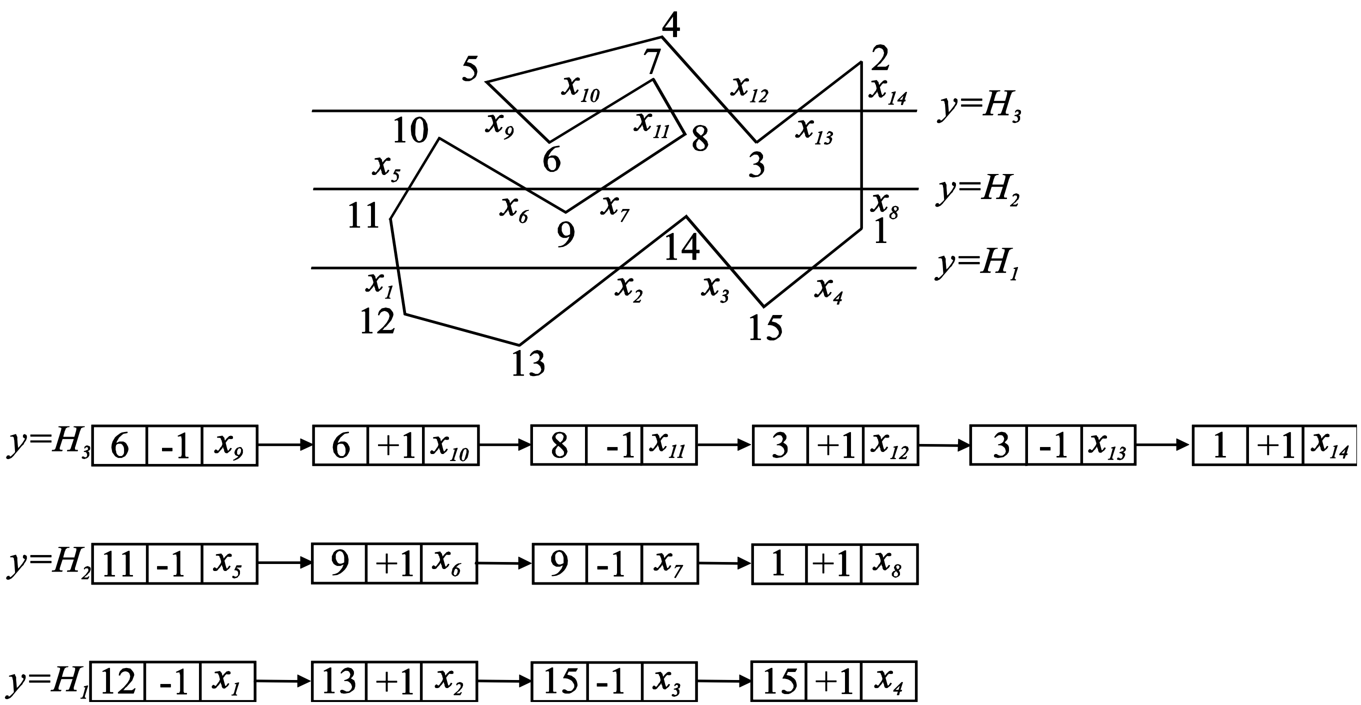


Updating the AET

Critical points Polygon Rasterization Algorithm

- Uses the local minima (critical points) explicitly in order to make ET redundant and to avoid its expensive creation

- An example polygon (above) and the contents of the AET for 3 scanlines (below)



Critical points Polygon Rasterization Algorithm

Algorithm:

1. Find and store the critical points of the polygon.
2. For every scanline y that intersects the polygon in an upward sweep

(a) For every critical point **c** $(c_x, c_y) \mid (y-1 < c_y \leqslant y)$ track the perimeter of the polygon in both directions starting at **c**. Tracking stops if scanline y is intersected or a local maximum is found. For every intersection with scanline y create an *AET* record $(v, \pm 1, x)$ containing the start vertex number v of the intersecting edge, the tracking direction along the perimeter of the polygon (−1 or +1 depending on whether it is clockwise or counterclockwise) and the x coordinate of the point of intersection.

(b) For every *AET* record that pre-existed step (a), track the polygon perimeter in the direction stored within it. If an intersection with scanline y is found, the record's start vertex number and intersection x coordinate are updated. If a local maximum is found the record is deleted from the *AET*.

(c) Sort the *AET* on x if necessary.

(d) Extract spans from the *AET* and set their pixels.

Triangle Rasterization Algorithm

- **Triangle**: simplest, planar, convex polygon
- Determine the pixels covered by a triangle → perform an inside test on all the pixels of the triangle's bounding box
- The inside test can be the evaluation of the 3 line functions defined by the triangle edges
- For each pixel p of the bounding box, if the 3 line functions give the same sign, then p is inside the triangle, otherwise outside
- For efficiency, the line functions are incrementally evaluated using their forward differences

Triangle Rasterization Algorithm (2)

Algorithm:

```
triangle1 ( vertex v0, v1, v2, colour c ) {
line l0, l1, l2;
float e0, e1, e2, e0t, e1t, e2t;
/* Compute the line coefficients (a,b,c) from the vertices */
mkline(v0, v1, &l0); mkline(v1, v2, &l1); mkline(v2, v0, &l2);
/* Compute bounding box of triangle */
bb_xmin = min(v0.x, v1.x, v2.x);
bb_xmax = max(v0.x, v1.x, v2.x);
bb_ymin = min(v0.y, v1.y, v2.y);
bb_ymax = max(v0.y, v1.y, v2.y);
/* Evaluate linear functions at (bb_xmin, bb_ymin) */
e0 = l0.a * bb_xmin + l0.b * bb_ymin + l0.c;
e1 = l1.a * bb_xmin + l1.b * bb_ymin + l1.c;
e2 = l2.a * bb_xmin + l2.b * bb_ymin + l2.c;
```

Triangle Rasterization Algorithm (3)

Algorithm (continued):

```
for (y=bb_ymin; y<=bb_ymax; y++) {
   e0t = e0; e1t = e1; e2t = e2;
   for (x=bb_xmin; x<=bb_xmax; x++) {
        if (sign(e0)==sign(e1)==sign(e2))
                setpixel(x,y,c);
        e0 = e0 + l0.a;
        e1 = e1 + l1.a;
        e2 = e2 + l2.a;
   }
   e0 = e0t + l0.b;
   e1 = e1t + l1.b;
   e2 = e2t + l2.b;
}
}
```

Triangle Rasterization Algorithm (4)

- If the bounding box is large, `triangle1` is wasteful
- Another approach: **Edge Walking**
  - 3 Bresenham line rasterization algorithms are used to walk the edges of the triangle
  - Trace is done per scanline by synchronizing the line rasterizers
  - The endpoints of a span of inside pixels are computed for every scanline that intersects the triangle and the pixels of the span are set
  - Special attention to special cases
- Simplicity of the above algorithms makes them ideal for hardware implementation

Area Filling Algorithms

- A simple approach is **flood fill**

Algorithm:

```
flood_fill ( polygon P, colour c ) {
point s;
draw_perimeter ( P, c );
s = get_seed_point ( P );
flood_fill_recur ( s, c );
}

flood_fill_recur ( point (x,y), colour fill_colour ); {
colour c;
c = getpixel(x,y); /* read current pixel colour */
if (c != fill_colour) {
    setpixel(x,y,fill_colour);
    flood_fill_recur((x+1,y), fill_colour ); flood_fill_recur((x-1,y),fill_colour );
    flood_fill_recur((x,y+1), fill_colour ); flood_fill_recur((x,y-1), fill_colour );
    }
}
```

Area Filling Algorithms (2)

- For 4 – connected areas the above 4 recursive calls are sufficient
- For 8 – connected areas 4 extra recursive calls must be added
  - `flood_fill_recur((x+1, y+1), fill_colour );`
  - `flood_fill_recur((x+1, y-1), fill_colour );`
  - `flood_fill_recur((x-1, y+1), fill_colour );`
  - `flood_fill_recur((x-1, y-1), fill_colour );`
- Basic problem its innefficiency

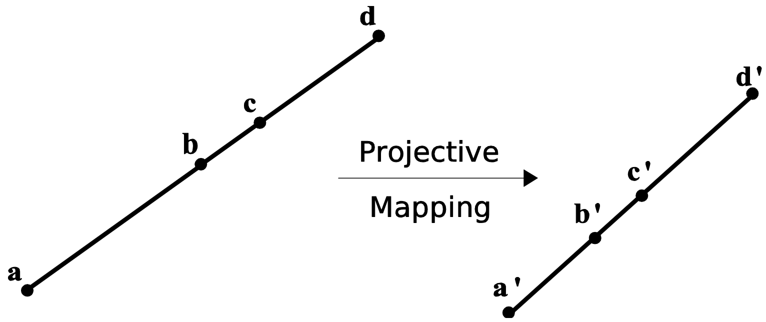
Perspective Correction

- The rasterization of primitives is performed in 2D screen space while the properties of primitives are associated with 3D object vertices
- The general projection transformation does not preserve ratios of distances $\rightarrow$ it is incorrect to linearly interpolate the values of properties in screen space
- **Perspective Correction** used to obtain the correct value at a projected point
- Based on the fact that projective transformations preserve cross ratios

Perspective Correction (2)

- Example:
  - Let **ad** be a line segment and **b** its midpoint in 3D space
  - Let **a’, d’, b’** be the perspective projections of the points **a, d, b**

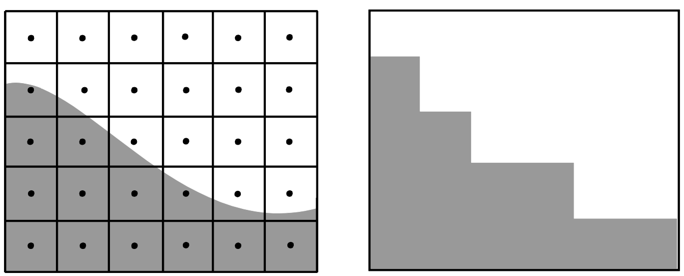
$$\left.\begin{array}{l} \dfrac{\;\frac{ac}{cd}\;}{\;\frac{ab}{bd}\;} = \dfrac{\;\frac{a'c'}{c'd'}\;}{\;\frac{a'b'}{b'd'}\;} \\[2ex] \dfrac{ab}{bd}=1 \quad , \quad \dfrac{a'b'}{b'd'}=q \end{array}\right\} \frac{ac}{cd}=\frac{a'c'}{qc'd'}$$

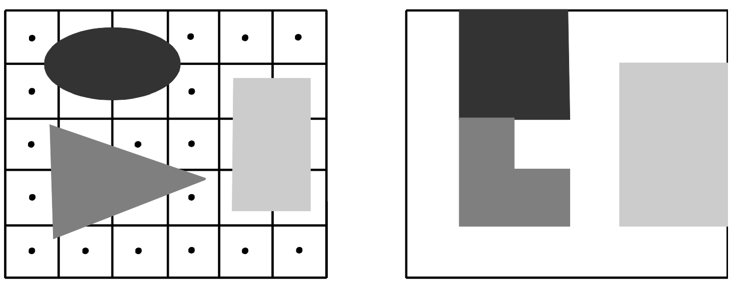


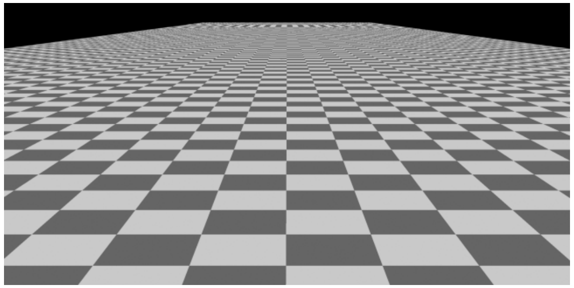
- Heckbert provides an efficient solution to perspective correction:
  - Perspective division of a property:
    - Let $[x, y, z, w, c]^T$ be the pre-perspective coordinates of a vertex, where c is the value of a property → $[x/w, y/w, z/w, c/w, 1/w]^T$ are the coordinates of the projected vertex

Spatial Anti-aliasing

- The primitive rasterization algorithms represent the pixel as a point
- Pixels are **not** mathematical points but have a small area → aliasing effects
- Aliasing effects:
  - jagged appearance of object silhouettes
  - improperly rasterized small objects
  - incorrectly rasterized detail







Anti-aliasing Techniques

- Anti-aliasing trades intensity resolution to gain spatial resolution

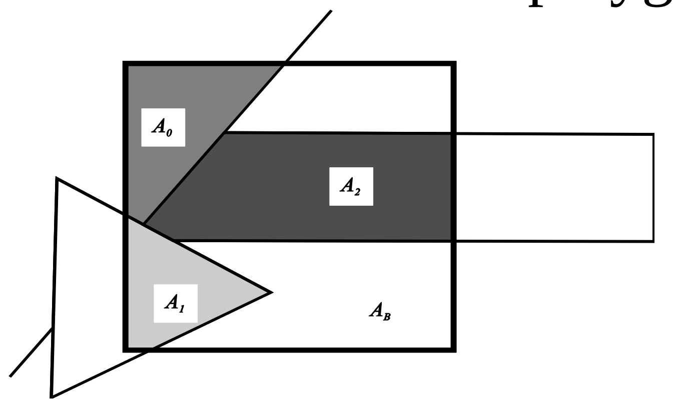
2 categories of anti-aliasing techniques:

- **Pre-filtering**:
  - extract high frequencies before sampling
  - treat the pixel as a finite area
  - compute the % contribution of each primitive in the pixel area
- **Post-filtering**:
  - extract high frequencies after sampling
  - increase sampling frequency
  - results are averaged down

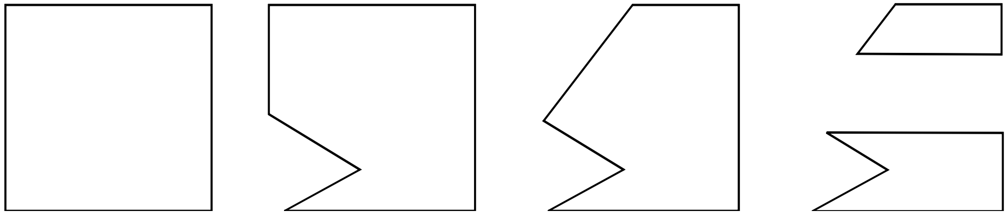
Pre-filtering Anti-aliasing Methods

Anti-aliased Polygon Rasterization: **Catmull's Algorithm**

- Consider each pixel as a square window
- Clip all overlapping polygons
- Estimate the visible area of each polygon as a % of the pixel



- A general polygon clipping algorithm is needed, such as Greiner-Horman (section 1.8.3)



Catmull's Algorithm

Algorithm:

1. Clip all polygons against the pixel window →

   $P_0 \ldots P_{n-1}$: the surviving polygon pieces
2. Eliminate hidden surfaces:

   (a) order by depth polygons $P_0 \ldots P_{n-1}$

   (b) clip against the area formed by subtracting the polygons from the (remaining) pixel window in depth order →

   $P_0 \ldots P_{m-1}$ $(m \leqslant n)$ the visible parts of polygons &

   $A_0 \ldots A_{m-1}$ their respective areas
3. Compute final pixel color: $A_0C_0 + A_1C_1 + \ldots + A_{m-1}C_{m-1} + A_BC_B$

   where C_i: the color of polygon I &

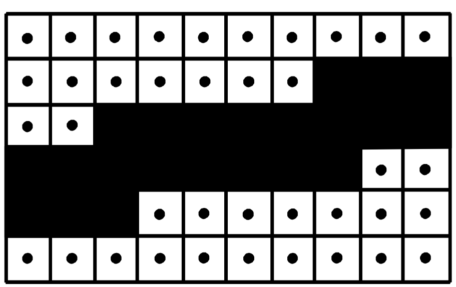
   A_B, C_B: background area & its color

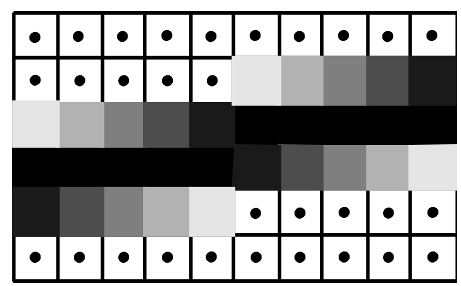
- Not practically viable:
  - Extraordinary computations
  - A polygon may not have constant color in a pixel (texture)

Pre-filtering Anti-aliasing Methods (2)

Anti-aliased Line Rasterization

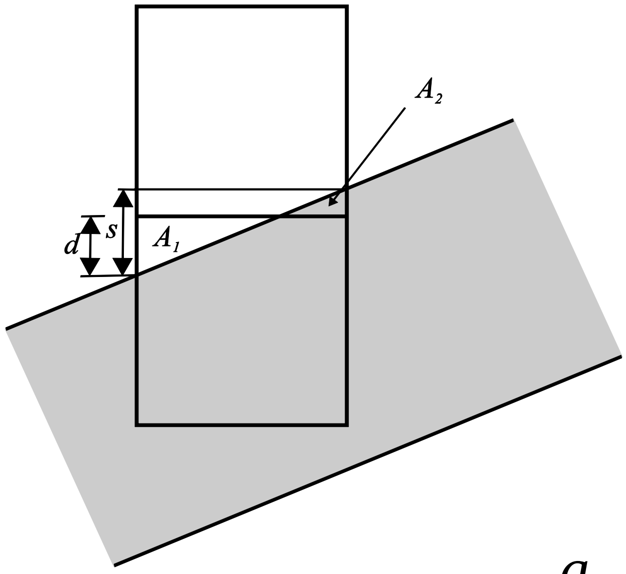
- Bresenham algorithm
  - uses binary decision to select the closest pixel to the mathematical path of the lines → jagged lines & polygon edges
- Lines must have certain width → modeled as thin parallelograms
  - binary decision is wrong
  - color value depends on the % of the pixel that is covered by the line





Anti-aliased Line Rasterization

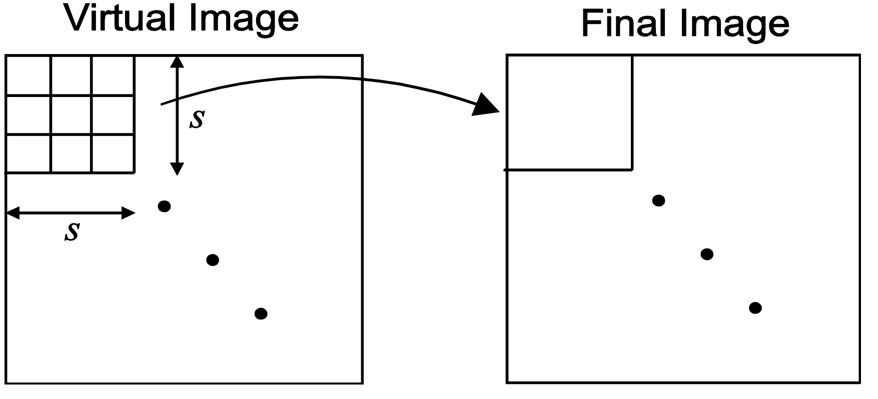
- An example:



- Line in the 1st octant with slope $s = -\frac{a}{b}$
- 2 pixels partially covered by the line
- Determine the portions of the triangles A_1 & A_2
- Color of the top pixel = color of line at a portion A_2
- Color of the bottom pixel = color of line at a portion $(1-A_1)$
- The areas of the triangles: $A_1 = \frac{d^2}{2s}$ $A_2 = \frac{(s-d)^2}{2s}$

Post-filtering Anti-aliasing Methods

- More than 1 sample per pixel → image at a higher resolution
- The results are averaged down to the resolution of the pixel grid
- Most common technique due to its simplicity
- An example:
  - to create an 1024 × 1024 image, take 3072 × 3072 samples
    - 9 samples per pixel (3 horizontally × 3 vertically)
  - 3 × 3 virtual image pixels correspond to 1 final image pixel
  - the final pixel's color is the average of the 9 samples



Post-filtering Algorithm

Algorithm:

1. The (continuous) image is sampled at s times the final pixel resolution (s horizontally × s vertically) creating a virtual image I_u.
2. The virtual image is low-pass filtered to eliminate the high frequencies that cause aliasing.
3. The filtered virtual image is re-sampled at the pixel resolution to produce the final image I_f

- Use s×s convolution filter h instead of averaging the s×s samples
- Steps:
  - Place the filter over the virtual image pixel
  - Compute the final image value: $I_f(i,j) = \sum_{p=0}^{s-1} \sum_{q=0}^{s-1} I_v(i*s+p, j*s+q) \cdot h(p,q)$
  - Move the filter

Post-filtering Algorithm (2)

- Examples of convolution filters:

3 x 3

1	2	1
2	4	2
1	2	1

5 x 5

1	2	3	2	1
2	4	6	4	2
3	6	9	6	3
2	4	6	4	2
1	2	3	2	1

7 x 7

1	2	3	4	3	2	1
2	4	6	8	6	4	2
3	6	9	12	9	6	3
4	8	12	16	12	8	4
3	6	9	12	9	6	3
2	4	6	8	6	4	2
1	2	3	4	3	2	1

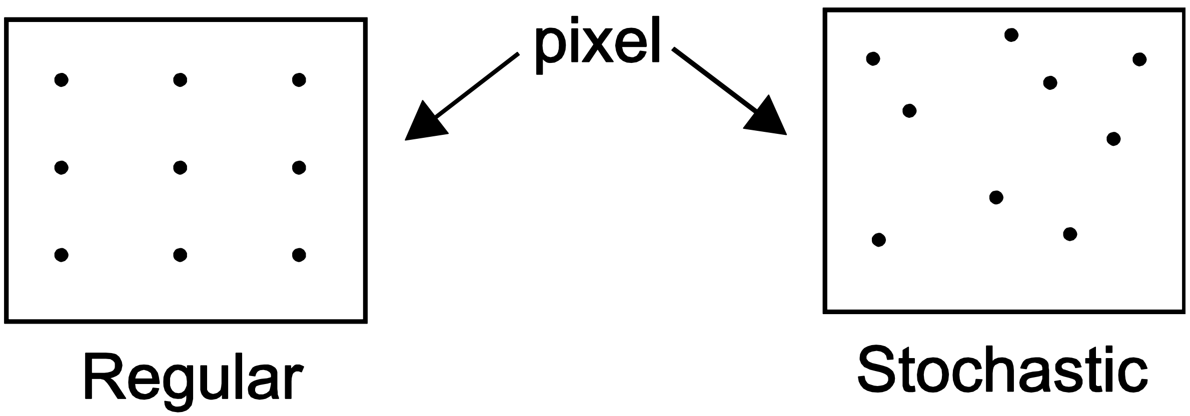
- To avoid color shifts, normalize:

$$\sum_{p=0}^{s-1} \sum_{q=0}^{s-1} h(p,q) = 1$$

- The larger the s is → better results
- Drawbacks:
  - ↑s → ↑image generation time & ↑ memory required
  - no matter how big s becomes, the aliasing problem will remain
  - not sensitive to image complexity → a lot of wasted computations

More Post-filtering Algorithms

- Adaptive post-filtering:
  - Increases the sampling rate where high frequencies exist
  - More complex algorithm
- Stochastic post-filtering:
  - Samples the continuous image at non-uniformly spaced positions
  - Aliasing effects are converted to noise (human eye ignores them)



2D Clipping Algorithms

- Avoid giving out-of-range values to a display device
- **Clipping object (window)**: display device usually modeled as rectangular parallelogram which defines the within-range values
- **Subject**: primitive of a modeled scene
- Generalization from 2D to 3D is relatively straightforward
- Subject relation to the clipping object
  - Subject entirely inside: rasterize it
  - Subject outside: do not rasterize
  - Subject intersects the clipping object: compute the intersection with a 2D clipping algorithm & rasterize the result

Point Clipping

- Point clipping is a trivial case:
  - is point (x, y) inside the clipping object ?
- If the clipping object is a rectangular parallelogram:
  - Exploit its opposite vertices (x_{min}, y_{min}), (x_{max}, y_{max})
- Inclusion Test:

  If $x_{min} \leq x \leq x_{max}$ & $y_{min} \leq y \leq y_{max}$

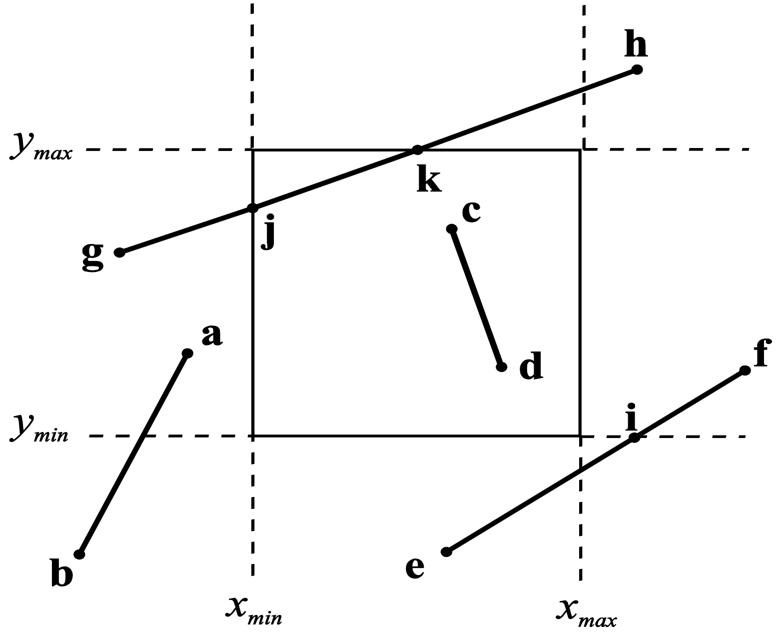
  Then the point is entirely inside and must be rasterized

  Else the point is entirely outside and must NOT be rasterized

Line Clipping - CS Algorithm

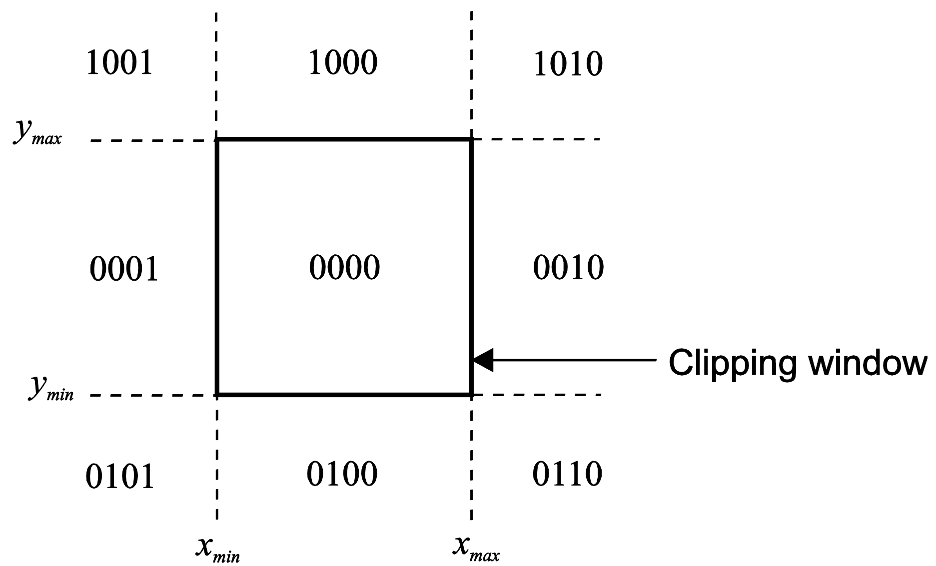
Cohen – Sutherland (CS) Algorithm

- Perform a low-cost test which decides if a line segment is entirely inside or entirely outside the clipping window
- For each non-trivial line segment compute its intersection with one of the lines defined by the window boundary
- Recursively apply the algorithm to both resultant line segments



Line Clipping - CS Algorithm (2)

- The plane of the clipping window is divided into 9 regions
- Each region is assigned a 4 – bit binary code
- The code bits are set according to the following rules:
  - **First Bit**: Set 1 for $y > y_{max}$, else set 0
  - **Second Bit**: Set 1 for $y < y_{min}$, else set 0
  - **Third Bit**: Set 1 for $x > x_{max}$, else set 0
  - **Fourth Bit**: Set 1 for $x < x_{min}$, else set 0



Line Clipping - CS Algorithm (3)

- Let the 4 – bit codes of the endpoints of a line segment be c_1, c_2
- Each endpoint is assigned a 4 – bit code according to the above rules
- Then the low-cost inclusion tests are:
  - If $c1 \vee c2 = 0000$

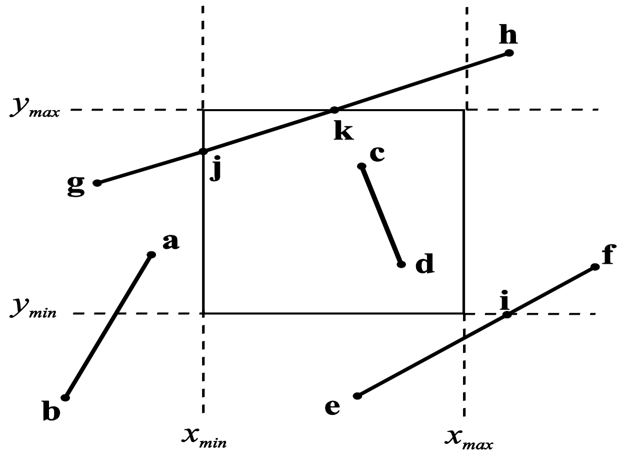
    Then the line segment is entirely inside
  - If $c1 \wedge c2 \neq 0000$

    Then the line segment is entirely outside

Line Clipping - CS Algorithm (4)

- Example :

Endpoint	Code	Endpoint	Code
a	0001	e	0100
b	0101	f	0010
c	0000	g	0001
d	0000	h	1010



- **ab** is entirely outside since $0001 \wedge 0101 \neq 0000$
- **cd** is entirely inside since $0000 \vee 0000 = 0000$
- For **ef** & **gh** the extent tests are not conclusive ⇨ compute the intersection points
- Intersect **ef** with line $y = y_{min}$ since the 2nd bit of the code is different at **e** & **f**
- Continue with the **if** line segment as the 2nd bit of the code of the **f** vertex has value 0 (inside)
- For **gh** compute one of the intersection points **k** & continue with **gk** which then computes the intersection **j** & recurses with a trivial inside decision for **jk**

Line Clipping - CS Algorithm (5)

Algorithm:

```
CS_Clip ( vertex p1, p2, float xmin, xmax, ymin, ymax ) {
int c1, c2;   vertex i;   edge e;
c1 = mkcode (p1);   c2 = mkcode (p2);
if ((c1 | c2) == 0)
   /* p1p2 is inside */
else if ((c1 & c2) != 0)
   /* p1p2 is outside */
else {
        e= /* window line with (c1 bit != c2 bit) */
        i = intersect_lines (e, (p1,p2));
        if outside (e, p1)
                CS_Clip(i, p2, xmin, xmax, ymin, ymax);
        else
                CS_Clip(p1, i, xmin, xmax, ymin, ymax);
      }
}
```

Line Clipping - Skala Algorithm

<u>Skala Algorithm:</u>

- Gain in efficiency over CS algorithm by classifying the vertices of the clipping window relative to the line segment being clipped
- A binary code c_i is assigned to each clipping window vertex $v_i = (x_i, y_i)$ as follows:
-
  - $$c_i = \begin{cases} 1, & l(x_i, y_i) \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

  where $l(x, y)$ is the function defined by the line segment to be clipped
- c_i indicates the side of the line segment that vertex v_i lies in

Line Clipping - Skala Algorithm (2)

- The codes are computed by taking the vertices in a consistent order around the clipping window (e.g. counterclockwise)

- A clipping window edge is intersected by the line segment for every change in the coding of the vertices (from 0 to 1 or from 1 to 0)

- A pre - computed table directly gives the clipping window edges intersected by the line segment from the code vector [c_0 ,c_1 ,c_2 ,c_3] and this replaces the recursive case of the CS algorithm

Line Clipping – LB Algorithm

Liang – Barsky (LB) Algorithm

- Solves the line clipping problem without using recursive calls
- Compared to CS algorithm, LB is more than 30% more efficient
- Can be easily extended to a 3D clipping object
- LB is based on the parametric equation of the line segment to be clipped from $\mathbf{p_1}(x_1, y_1)$ to $\mathbf{p_2}(x_2, y_2)$:

$$\mathbf{P} = \mathbf{p_1} + t\,(\mathbf{p_2} - \mathbf{p_1}), \quad t \in [0, 1]$$

or

$$x = x_1 + t\,\Delta x\,, \quad y = y_1 + t\,\Delta y$$

where

$$\Delta x = x_2 - x_1\,, \; \Delta y = y_2 - y_1$$

Line Clipping – LB Algorithm (2)

- For the part of the line segment that is inside the clipping window:

$$x_{min} \le x_1 + t\,\Delta x \le x_{max}\ ,$$
$$y_{min} \le y_1 + t\,\Delta y \le y_{max}$$

or

$$-t\,\Delta x \le x_1 - x_{min}\ ,$$
$$t\,\Delta x \le x_{max} - x_1\ ,$$
$$-t\,\Delta y \le y_1 - y_{min}\ ,$$
$$t\,\Delta y \le y_{max} - y_1$$

Line Clipping – LB Algorithm (3)

- The above inequalities have the common form:

  $t\,p_i \leq q_i$,

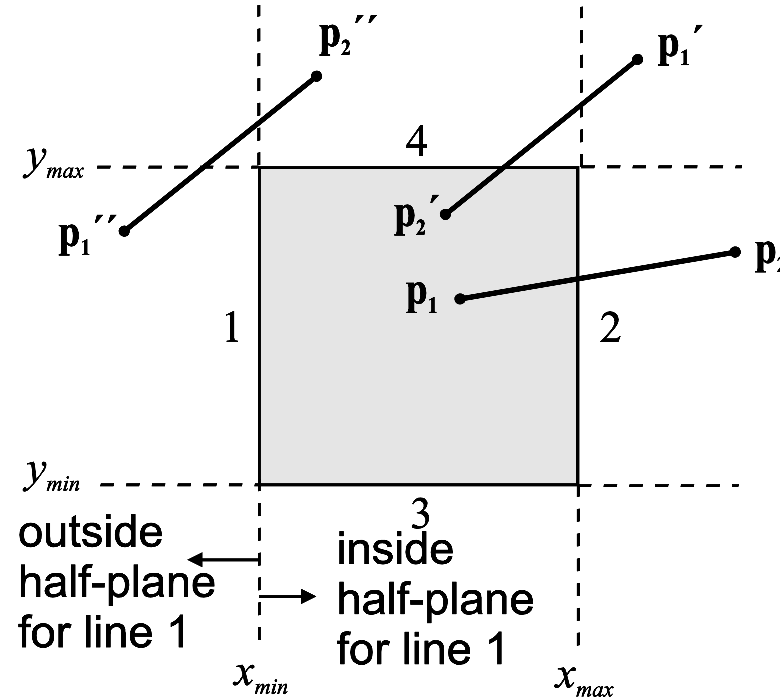
  where

  $p_1 = -\Delta x$, $q_1 = x_1 - x_{min}$

  $p_2 = \Delta x$, $q_2 = x_{max} - x_1$

  $p_3 = -\Delta y$, $q_3 = y_1 - y_{min}$

  $p_4 = \Delta y$, $q_4 = y_{max} - y_1$



Line Clipping – LB Algorithm (4)

- Notice the following:
  - If $\mathbf{p_i} = 0$ the line segment is parallel to the window edge i and the clipping problem is trivial
  - If $\mathbf{p_i} \neq 0$ the parametric value of the point of intersection of the line segment with the line defined by window edge i is

    $$\mathbf{t_i = q_i / p_i}$$

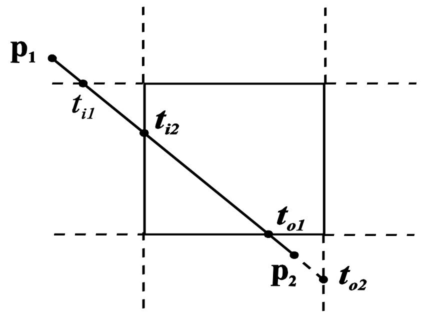
  - If $\mathbf{p_i} < 0$ the directed line segment is incoming with respect to window edge i
  - If $\mathbf{p_i} > 0$ the directed line segment is outgoing with respect to window edge i

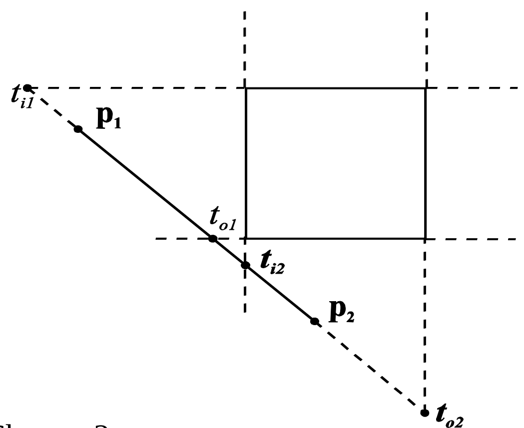
Line Clipping – LB Algorithm (5)

- Therefore $t_{\mathbf{in}}$ and $t_{\mathbf{out}}$ can be computed as:

$$t_{in} = \max(\{\frac{q_i}{p_i} \mid p_i < 0,\ i:1..4\} \cup \{0\}),\quad t_{out} = \min(\{\frac{q_i}{p_i} \mid p_i > 0,\ i:1..4\} \cup \{1\})$$

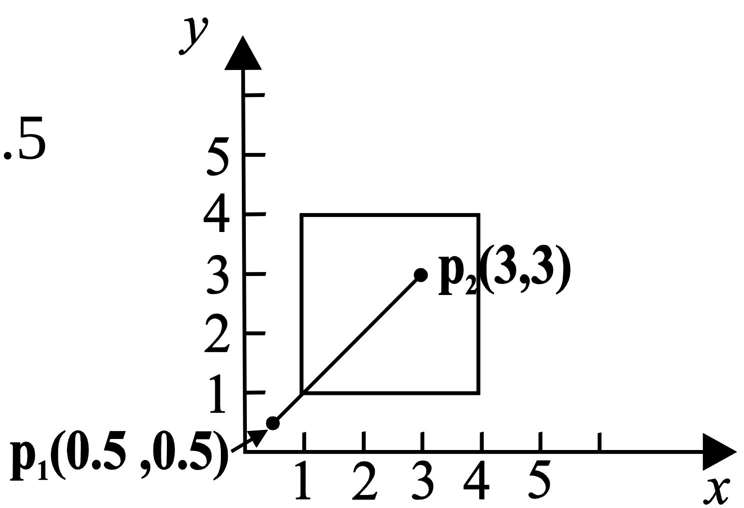
- Sets {0}, {1} clamp the starting and ending parametric values at the end points of the line segment
- If $\mathbf{t_{in}} \le \mathbf{t_{out}}$, the values $\mathbf{t_{in}}$ and $\mathbf{t_{out}}$ are plugged into parametric line equation to get the actual starting – ending points of the clipped segment
- Otherwise there is no intersection with the clipping window





Line Clipping – LB Algorithm (6)

<u>LB example:</u>



- Compute: $\Delta x = 2.5$ and $\Delta y = 2.5$
- Compute: p_1=-2.5, q_1=-0.5
  p_2=2.5, q_2=3.5
  p_3=-2.5, q_3=-0.5
  p_4=2.5, q_4=3.5.
- Compute: $t_{in} = \max(\{\frac{q_1}{p_1}, \frac{q_3}{p_3}\} \cup \{0\}) = 0.2$, $t_{out} = \min(\{\frac{q_2}{p_2}, \frac{q_4}{p_4}\} \cup \{1\}) = 1$
- Since $\mathbf{t_{in}} < \mathbf{t_{out}}$ compute endpoints $\mathbf{p_1}'(x_1', y_1')$, $\mathbf{p_2}'(x_2', y_2')$ of the clipped line segment using the parametric equation:

$$x_1' = x_1 + t_{in}\Delta x = 0.5 + 0.2 \cdot 2.5 = 1$$

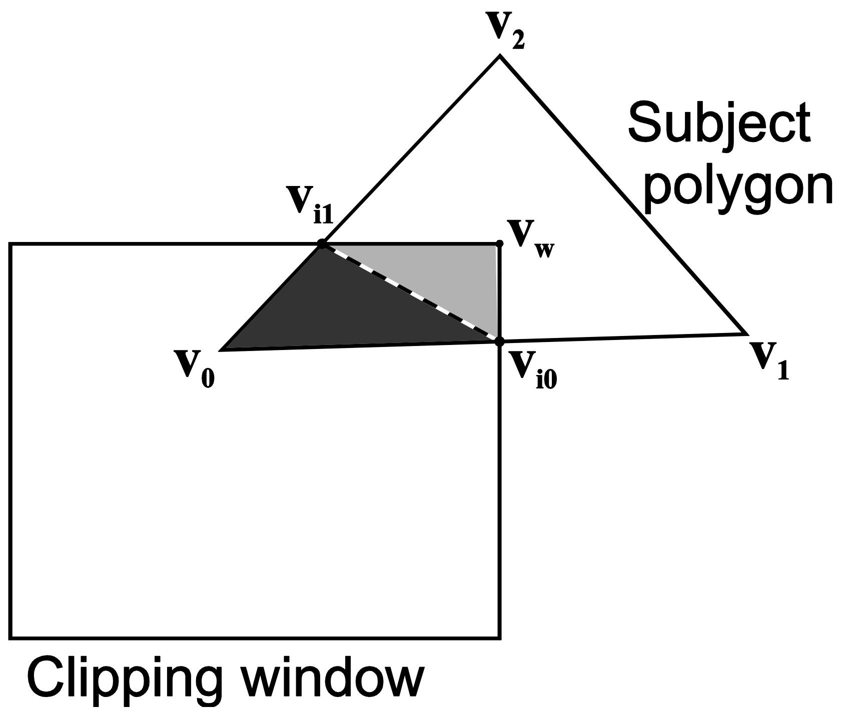
$$y_1' = y_1 + t_{in}\Delta y = 0.5 + 0.2 \cdot 2.5 = 1$$

$$x_2' = x_1 + t_{out}\Delta x = 0.5 + 1 \cdot 2.5 = 3$$

$$y_2' = y_1 + t_{out}\Delta y = 0.5 + 1 \cdot 2.5 = 3$$

Polygon Clipping

- In 2D polygon clipping the subject and clipping object are both polygons (**subject polygon**, **clipping polygon**)
- Why is polygon clipping important ?

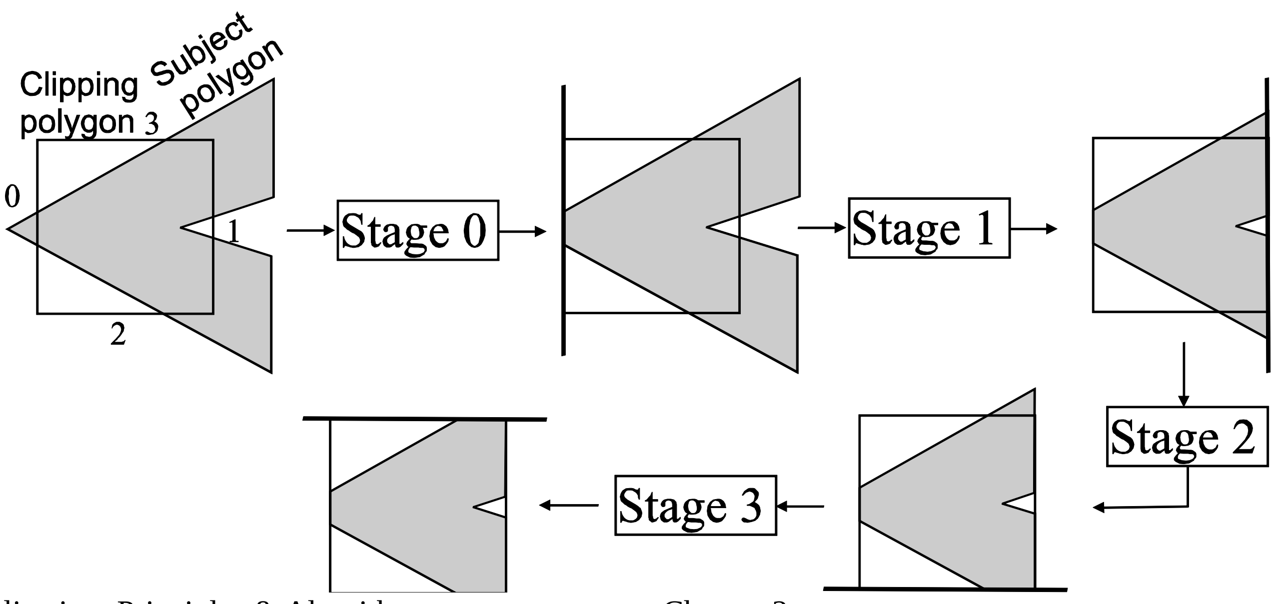


- Polygon clipping cannot be regarded as multiple line clipping

Polygon Clipping – SH Algorithm

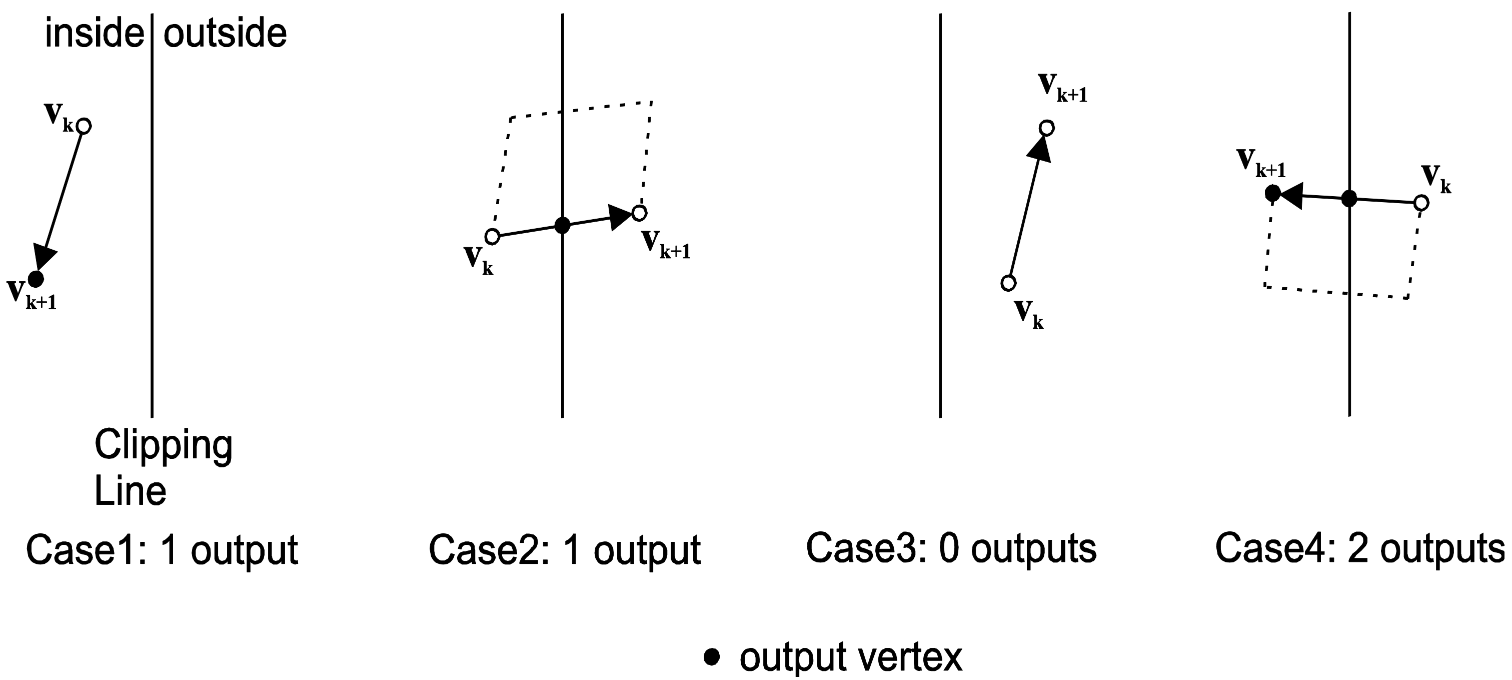
Sutherland – Hodgman (SH) Algorithm:

- Clips an arbitrary subject polygon against a **convex** clipping polygon
- Has *m* pipeline stages which correspond to the *m* edges of the clipping polygon
- Stage i | i: 0…*m*-1 clips the subject polygon against the line defined by edge i of the clipping polygon
- The input to stage i | i: 1…*m*-1 is the output of stage i-1
- Polygon is restricted to be convex



Polygon Clipping – SH Algorithm (2)

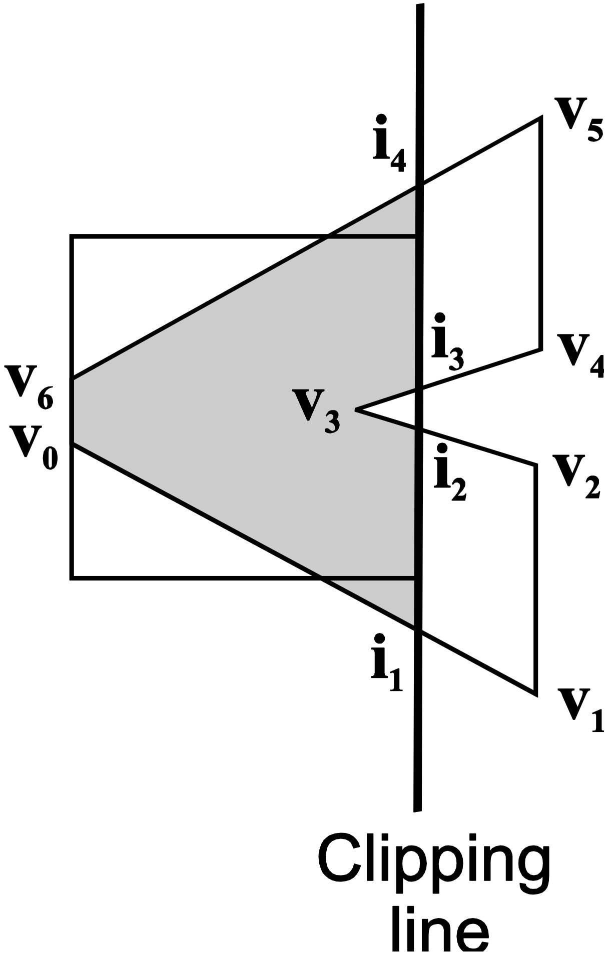
- For each stage of the SH algorithm there are the following 4 relationships between a clipping line and an object polygon edge $v_k v_{k+1}$



Polygon Clipping – SH Algorithm (3)

- Example of the 1st stage of the SH algorithm:

v_k	v_{k+1}	Case	Output
v_0	v_1	2	i_1
v_1	v_2	3	-
v_2	v_3	4	i_2, v_3
v_3	v_4	2	i_3
v_4	v_5	3	-
v_5	v_6	4	i_4, v_6
v_6	v_0	1	v_0



Polygon Clipping – SH Algorithm (4)

- ***Algorithm:***

```
polygon SH_Clip ( polygon C, S ) { /*C must be convex*/
   int i, m;
   edge e;
   polygon InPoly, OutPoly;
   m = getedgenumber(C);
   InPoly = S;
   for (i=0; i<m; i++) {
        e = getedge(C,i);
        SH_Clip_Edge(e,InPoly,OutPoly);
        InPoly = OutPoly
        }
   return OutPoly
}
```

Polygon Clipping – SH Algorithm (5)

- ***Algorithm:***

```
SH_Clip_Edge ( edge e, polygon InPoly, OutPoly ) {
    int k, n; vertex vk, vkplus1, i;
    n = getedgenumber(InPoly);
    for (k=0; k<n; k++) {
        vk = getvertex(InPoly,k); vkplus1=getvertex(InPoly,(k+1) mod n);
        if (inside(e, vk) and inside(e, vkplus1))
                /* Case 1 */
                putvertex(OutPoly,vkplus1)
        else if (inside(e, vk) and !inside(e, vkplus1)) {
                /* Case 2 */
                i = intersect_lines(e, (vk,vkplus1)); putvertex(OutPoly,i)
        }
        else if (!inside(e, vk) and !inside(e, vkplus1))
                /* Case 3 */
        else {
                /* Case 4 */
                i = intersect_lines(e, (vk,vkplus1)); putvertex(OutPoly,i);
                putvertex(OutPoly,vkplus1)
        }
    }
}
```

Polygon Clipping – SH Algorithm (6)

- The complexity of SH algorithm is $O(mn)$ where m and n are the numbers of vertices of the clipping and subject polygons respectively

- No complex data structures or operations are required so the SH algorithm is quite efficient

- The SH algorithm is appropriate for hardware implementation since the clipping polygon, in general, is constant

Polygon Clipping – GH Algorithm

Greiner – Hormann Algorithm

- Suitable for general clipping polygons (*C*) and subject polygons (*S*)
- The polygons can be arbitrary closed polygons, even self intersecting
- The complexity of step 1 and 2 is *O(mn)* where m and n are the numbers of vertices of the *C* and *S* polygon respectively
- The overall complexity of the GF algorithm is *O(mn)*
- In practice, the complex data structures used in GF algorithm makes it less efficient than the SH algorithm

Polygon Clipping – GH Algorithm (2)

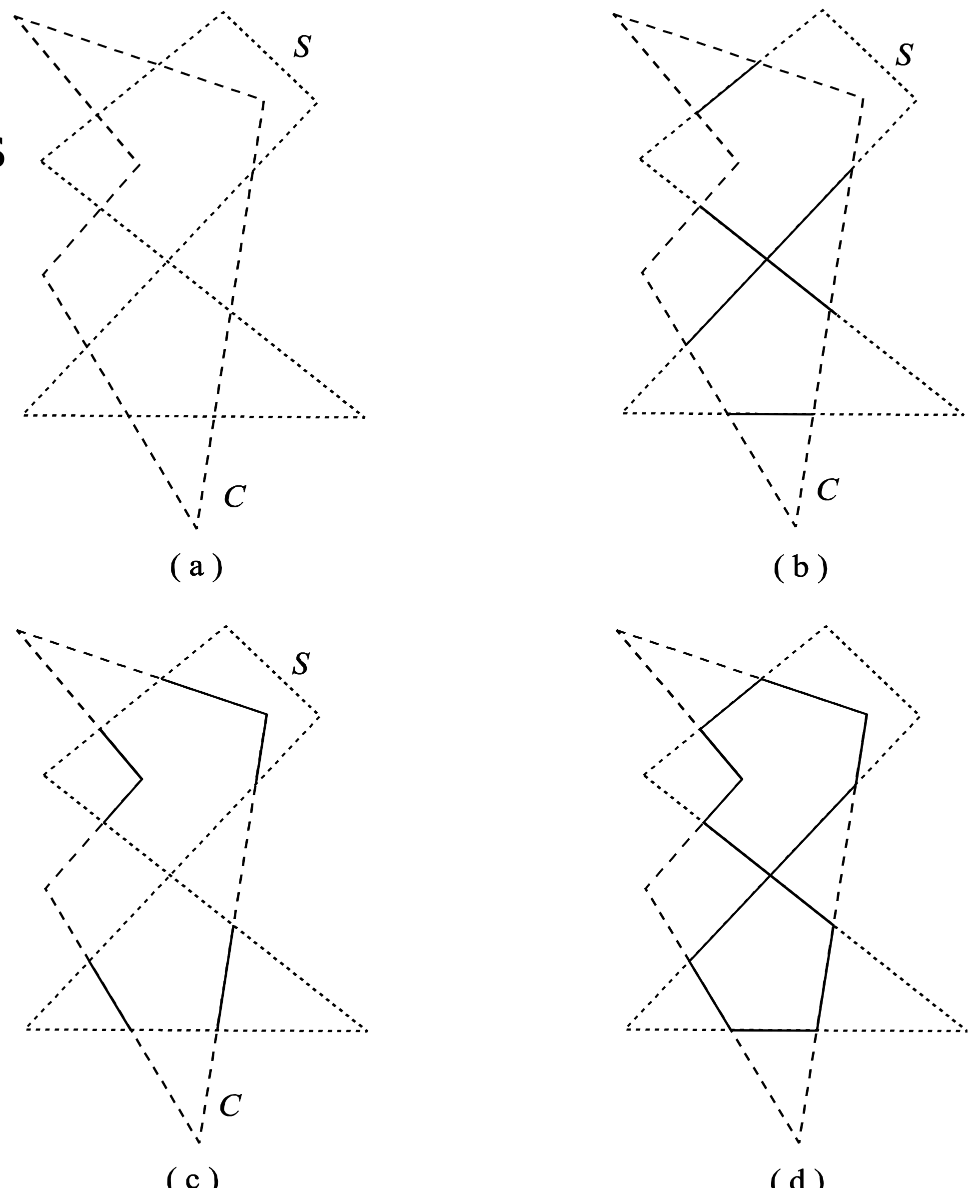
- GH algorithm is based on the winding number test for point *p* in polygon *P*, symbolically → $\omega(P, p)$
- $\omega(P, p)$ does not change so long as the topological relation of the point *p* and the polygon *P* remains constant
- If *p* crosses *P* the $\omega(P, p)$ is incremented or decremented
- If $\omega(P, p)$ is odd then *p* is inside *P*, otherwise it is outside

Polygon Clipping – GH Algorithm (3)

- The 3 steps of the GH algorithm:
  1. Trace the perimeter *S* starting from a vertex v_{s0}. An imaginary stencil toggles between *on* and *off* state every time the perimeter of *C* is crossed. Its initial state is *on* if v_{s0} is inside *C* and *off* otherwise. It thus computes the part of the perimeter of *S* that is inside *C*
  2. As step 1 but reverse the roles of *S* and *C*. The part of the perimeter of *C* that is inside *S* is thus computed
  3. The union of the results of steps 1 and 2 is the result of clipping *S* against *C* (or equivalently *C* against *S*)

Polygon Clipping – GH Algorithm (4)

- GH algorithm example:
  - (a) The initial *S*, *C* polygons
  - (b) After step 1 of GH
  - (c) After step 2 of GH
  - (d) The final result



Polygon Clipping – GH Algorithm (5)

- GH algorithm computes the intersection of the areas of 2 polygons, $\mathbf{C} \cap \mathbf{S}$
- It easily generalizes to compute $\mathbf{C} \cup \mathbf{S}$, **C** – **S** and **S** – **C** by changing the initial states of the stencils for **S** and **C**
- Obviously there are 4 possible combinations of the initial state
- These generalizations are not useful for the clipping problem