# Graphics & Visualization

**Chapter 6**

# *MODEL REPRESENTATION AND SIMPLIFICATION*

# Introduction

- 3D scenes in graphics are composed of various shapes and structures:
  - Geometric primitives (spheres)
  - Free – form surfaces mathematically defined (NURBS patches)
  - Arbitrary surfaces mathematically undefined (surface of a scanned object)
  - Volume objects, where the internal structure of the object is equally important to its boundary surface (human organ)
  - Fuzzy objects (smoke)
- *Models* are approximate representations of the actual objects, constructed to retain many of the properties of the object
- Models are amenable to the manipulation required by graphics algorithms

# Introduction (2)

- *Polygonal models* are the most common representation for surfaces
- Information contained in models produced is growing constantly
- Mainstream graphics applications often require or benefit from less detailed models
- *Model simplification* reduces the amount of information present in a model, without significantly sacrificing the quality of the representation

# Overview of Model Forms

- There are two main categories of models:
  - *Surface representation* (or *boundary representation* or *b-rep*)
    - Represents only the surface of an object
  - *Volume representation* (or *space subdivision*)
    - Represents the whole volume that a closed object occupies
- Surface representations are used more frequently because:
  - Many objects are not closed → volume representation is not applicable
  - Majority of objects are not transparent → space and processing power is saved by only representing their surface, which determines their appearance
- Volume representations are used:
  - When displaying semi-transparent objects
  - When displaying objects whose internal structure is of interest
  - As auxiliary structures in general graphics algorithms

# Overview of Model Forms (2)

- Some models cannot be easily classified into these two categories:
  - *Constructive Solid Geometry (CSG) models* represent an object by combining geometric primitives
  - Amorphous objects and phenomena may be modeled as point clouds or by aggregating simple surface or volume primitives
- Surface models are classified:
  - To those that have some mathematical description such as:
    - Geometric primitives
    - NURBS surfaces
    - Subdivision surfaces
    - General parametric surfaces
  - And those that do not have such a mathematical description:
    - Consist of a set of points and a set of planar (usually) polygons constructed with these points as vertices → *polygonal models*

# Overview of Model Forms (3)

- Comparing the two surface model forms:
  - Mathematical models:
    - Are usually exact representations of the respective objects
    - Allow computations on object (e.g. normal vector) to be performed exactly
    - Are limited to specific kind of objects
    - Cannot describe arbitrary shapes
  - Polygonal models:
    - Are approximations of the original objects
    - Albeit very precise ones if enough vertices are used
    - Are the most general
    - Even mathematical representations are usually rendered in a “discrete” form as polygonal models

# Overview of Model Forms (4)

- Polygon models may consist of polygons of any number of vertices. In practice:
  - Quadrilaterals
  - Triangles
- Quadrilateral models:
  - Are naturally generated when rasterizing parametric surfaces
  - Unfortunately, a quadrilateral in 3D is not necessary planar:
    - restricts the shape and flexibility of the model
    - Even if planarity is enforced, the computations are difficult
- Triangle models:
  - A triangle is always planar
  - Any polygon may be triangulated efficiently → a triangle model can be generated from any other polygonal model → triangle models (or *triangle meshes*) are almost always preferred for any application involving polygonal models

# Overview of Model Forms (5)

- Polygon models are generalized to polyhedral models for volume representation
- Most basic polyhedral primitive is the *tetrahedron* → *tetrahedral meshes* are the most general and flexible representation of volume models
- Models consisting of parallelepipeds are abundant, mainly as the outcome of space subdivision processes that use rectangular grids
- Constituent parallelepipeds are called *voxels* (volume elements)
- Hierarchical volume representations (octrees, BSP trees) are also used

- We will focus on polygonal models

# Properties of Polygonal Models

- A surface model is a *2-manifold* if every point on the surface has a neighborhood homeomorphic to an open disk (circle interior)
  - Even though the surface exists in 3D space, it is topologically flat when examined closely in a small area around any given point
- On a manifold surface:
  - Every edge is shared by exactly 2 faces
  - Around each vertex exists a closed loop of faces
- A surface model is a *manifold with boundary* if every point on the surface has a neighborhood homeomorphic to a half disk
- On a manifold with a boundary:
  - Some edges (those on the boundary) belong to exactly one face
  - Around some vertices (those on the boundary) the loop of faces is open
- For the usual, 3D surfaces, a manifold surface without boundary is a *closed* surface

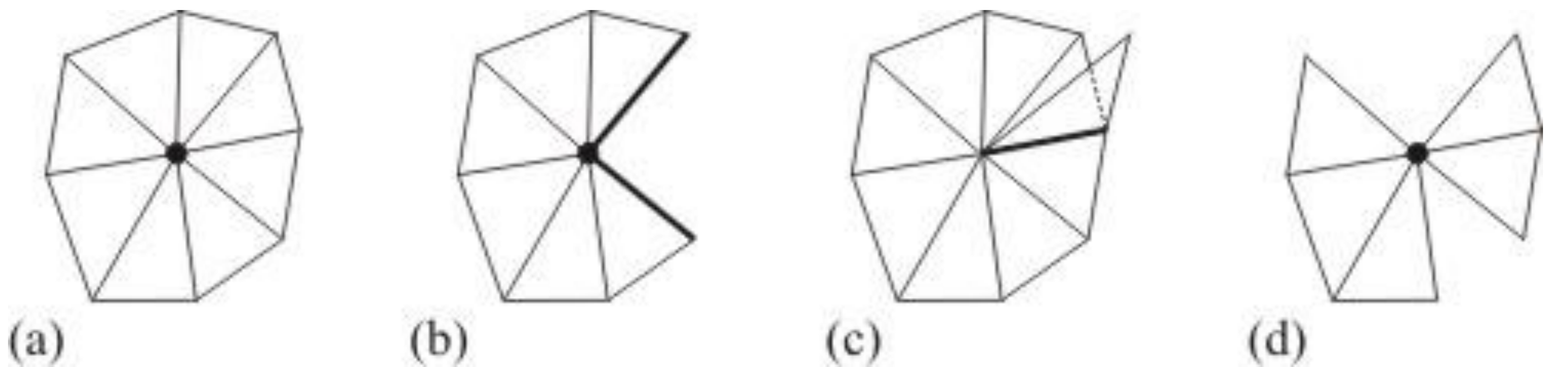
# Properties of Polygonal Models (2)

(a) Part of manifold surface

(b) Boundary vertex of a manifold surface with boundary

(c) Non manifold edge

(d) Non manifold boundary vertex

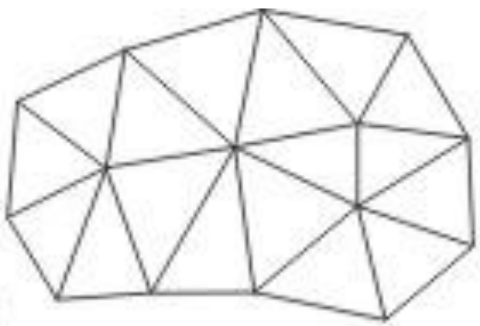



- A surface model is a *simplicial complex* if its constituting polygons meet only along their edges, and the edges of the model intersect only at their endpoints

# Properties of Polygonal Models (3)

- A surface model is a *simplicial complex* if its constituting polygons meet only along their edges, and the edges of the model intersect only at their endpoints

- (a) Simplicial triangle mesh (b) non simplicial triangle mesh

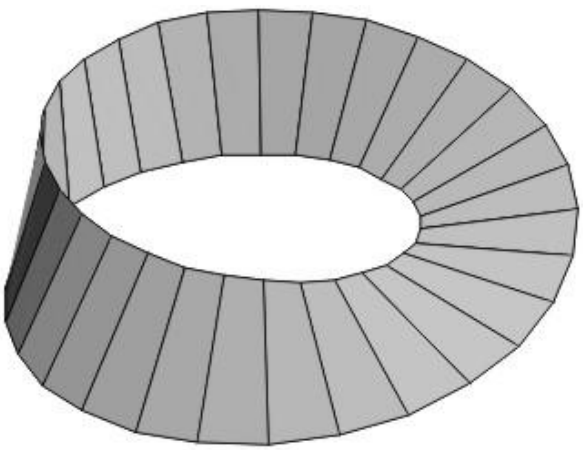


(a)

(b)

# Properties of Polygonal Models (4)

- *Orientable surface*: surface that has 2 “sides”, like a sheet of paper
- Most of the surfaces are orientable
- On closed orientable surfaces the “external” and “internal” portions of the surface are distinguishable
- By convention, the normal vector of a closed orientable surface points towards “outside”
- The Moebius strip is a non – orientable surface:



# Properties of Polygonal Models (5)

- Closed manifold models homeomorphic to a sphere satisfy Euler’s formula:

$$V – E + F = 2$$

where:

$$\left.\begin{array}{l}\text{V: \# of vertices}\\ \text{E: \# of edges}\\ \text{F: \# of faces}\end{array}\right\}\quad\text{of the model}$$

# Properties of Polygonal Models (6)

- For a closed triangular model the formula reveals:
  - That the number of triangles of the model is almost twice the number of its vertices
  - That the average number of triangles around each vertex is 6
- Euler’s formula has been generalized for arbitrary manifold models:

$$V - E + F = 2 - 2G$$

  where G is the genus of the model

- The genus of a model can be considered as the number of the penetrating holes of the model:
  - Torus has genus 1
  - Double torus has genus 2, and so on

# Data Structures for Polygonal Models

- Several different data structures have been proposed for representation of polygon models; they differ:
  - In the type of polygon models that they are able to represent
  - In the amount and type of information that they capture directly about the model
  - In other information that can or cannot be derived indirectly from them about the model
- Useful information for several graphics operations is:
  - *Topological information*: whether the model is manifold, closed, has a boundary or holes
  - *Adjacency information*: neighboring faces of given edge and face, edges and faces around a given vertex, the boundary of an open model
  - *Attributes attached to the model*: normal vector, colors, material properties, texture coordinates

# Data Structures for Polygonal Models (2)

- Most primitive data structures that were used:
  - *Explicit list of edges* — contain for each edge/face of the model the coordinates of its vertices
  - *Explicit list of faces*

**EXAMPLE:**

- For the tetrahedron beside it holds:
  - List of edges:

$e_0=((x_0,y_0,z_0),(x_1,y_1,z_1))$, $e_3=((x_1,y_1,z_1),(x_2,y_2,z_2))$,

$e_1=((x_0,y_0,z_0),(x_2,y_2,z_2))$, $e_4=((x_1,y_1,z_1),(x_3,y_3,z_3))$,

$e_2=((x_0,y_0,z_0),(x_3,y_3,z_3))$, $e_5=((x_2,y_2,z_2),(x_3,y_3,z_3))$

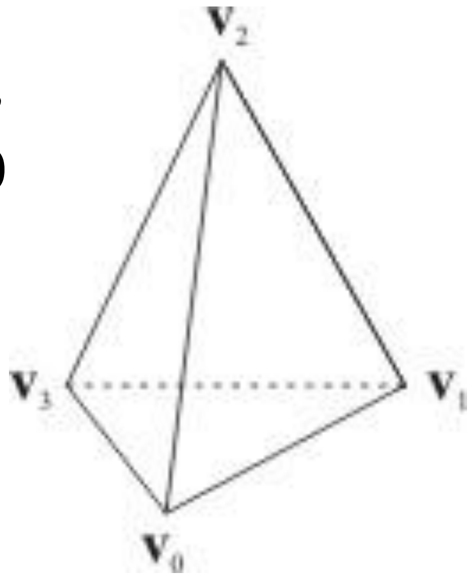
  - List of faces:

$f_0=((x_3,y_3,z_3),(x_2,y_2,z_2),(x_1,y_1,z_1))$,

$f_1=((x_2,y_2,z_2),(x_3,y_3,z_3),(x_0,y_0,z_0))$,

$f_2=((x_1,y_1,z_1),(x_0,y_0,z_0),(x_3,y_3,z_3))$,

$f_3=((x_0,y_0,z_0),(x_1,y_1,z_1),(x_2,y_2,z_2))$



# Data Structures for Polygonal Models (3)

- List of edges:
  - Is not a b-rep
  - Does not specify the faces of the model
  - Faces must be inferred from the edge data → may lead to ambiguities
- List of faces:
  - The coordinates of each vertex are repeated for each edge or face containing it → wastes space
  - Provides no information on the adjacency of the faces and edges
  - Common vertices can only be detected by comparing coordinates → numerical accuracy problems may interfere → computing adjacency can be problematic

# Data Structures for Polygonal Models (4)

- Several of the above drawbacks are addressed by the *indexed list of faces*:
  - Contains a list of the vertices of the model and a list of its faces
  - The vertices of its faces are given as references to the list of vertices
- For instance, the previous tetrahedron is represented as:

$$\mathbf{v}_0=(x_0,y_0,z_0),\quad f_0=(\mathbf{v}_3,\mathbf{v}_2,\mathbf{v}_1),$$
$$\mathbf{v}_1=(x_1,y_1,z_1),\quad f_1=(\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_0),$$
$$\mathbf{v}_2=(x_2,y_2,z_2),\quad f_2=(\mathbf{v}_1,\mathbf{v}_0,\mathbf{v}_3),$$
$$\mathbf{v}_3=(x_3,y_3,z_3),\quad f_3=(\mathbf{v}_0,\mathbf{v}_1,\mathbf{v}_2)$$

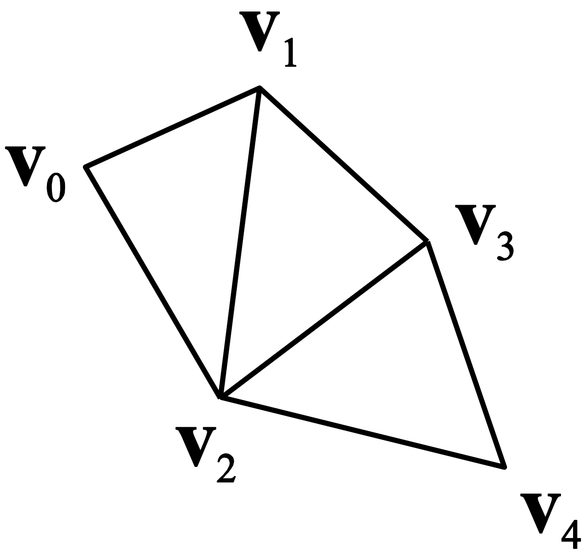
- To represent orientable models, using indexed list of faces, it is customary to list the vertices of all faces either clockwise or counterclockwise → easier to make computations on the model

# Data Structures for Polygonal Models (5)

- Indexed list of faces:
  - Can represent any kind of polygon model
  - Permits direct modifications to the positions of the vertices of the model
  - Edges of the model are straightforward to discover but they are repeated for each polygon that uses them
  - Processing is required in order to generate a valid list of unique edges
  - Does not provide adjacency information although the data it contains is sufficient to compute it
- Specifically for triangle models, neighboring triangles are handled more efficiently as *triangle strips* or *triangle fans*, in order to minimize data duplication

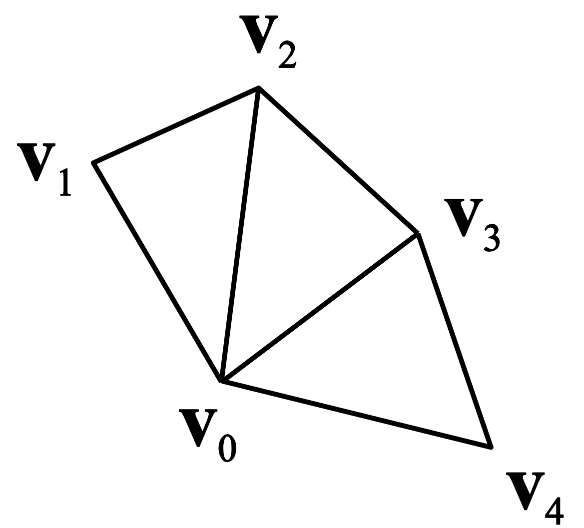
# Data Structures for Polygonal Models (6)

- Specifically for triangle models, sets of neighboring triangles are handled more efficiently as *triangle strips* or *triangle fans*, in order to minimize data duplication




Triangle strip

$(\mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$




Triangle fan

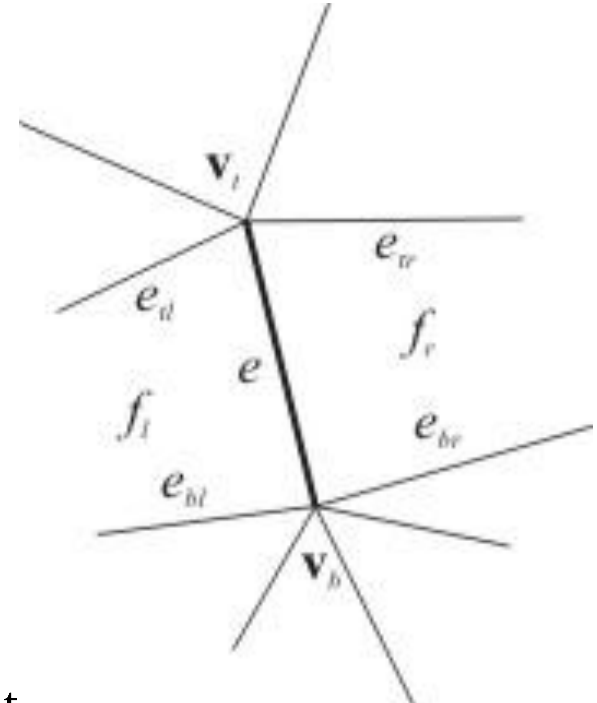
$(\mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4)$

# Data Structures for Polygonal Models (7)

- Indexed list of faces can be combined with other indexed data for the attributes of the model bound to either vertices or faces:
  - i.e. color

- More advanced data structures can capture directly some adjacency information and allow for easy derivation of more adjacency relations
- These data structures are indexed, contain at least a list of vertices and deal with manifold models

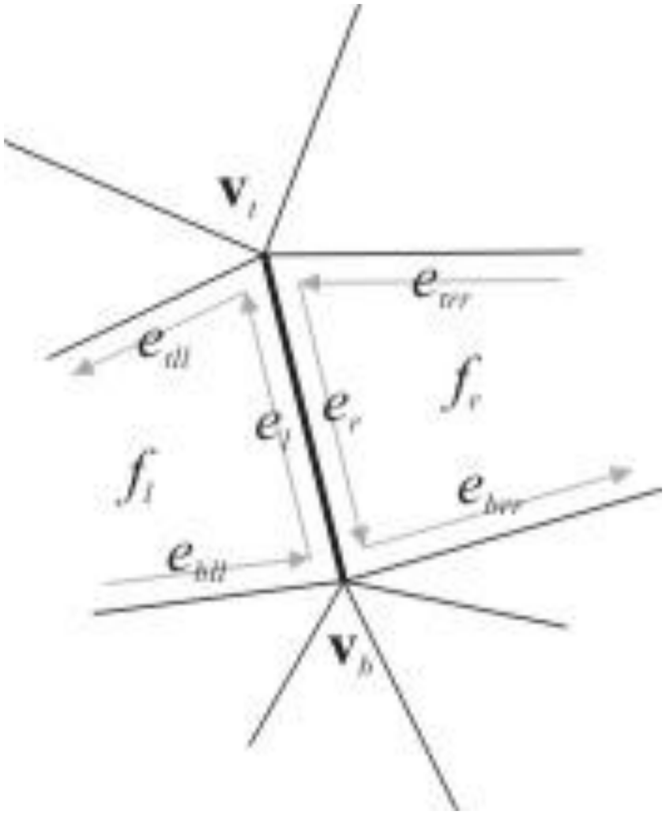
# Data Structures for Polygonal Models (8)




- *Winged-edge*, is one such data structure:
  - Central node of information is the edge
  - Each edge stores references:
    - To its 2 vertices
    - To its 2 adjacent faces
    - To its 4 neighboring edges
  - For each vertex a reference to one of its incident edges is stored
  - For each face a reference to one of its edges is stored
  - Possible to "navigate" in the topology of the model and compute adjacent queries efficiently
  - Winged-edge can be modified in order to represent some types of non manifold models

# Data Structures for Polygonal Models (9)

- *Half-edge* data structure is similar to winged-edge representation, but uses oriented edges:
  - Each edge is “decomposed” into 2 half-edges
  - Each half-edge stores references:
    - To its start and end vertex
    - To its adjacent face
    - To its 2 neighboring half-edges along the adjacent face
    - To its opposite half-edge
  - Half-edge data structure is more efficient than winged-edge for several adjacency queries



# Data Structures for Polygonal Models (10)

- *Quad-edge* data structure is similar to the above representations:
  - Its implementation is more sophisticated
  - Can be used to compute adjacency queries efficiently
  - Can represent simultaneously a manifold model and its *dual*
    - Dual of a model is constructed by rotating edges by $90^{o}$, replacing the vertices with faces and vice versa

      i.e. dual of a tetrahedron is a tetrahedron

      dual of a cube is an octahedron and vice versa
  - Useful in the context of *computational geometry*, the algorithmic study of geometric problems

# Polygonal Model Simplification

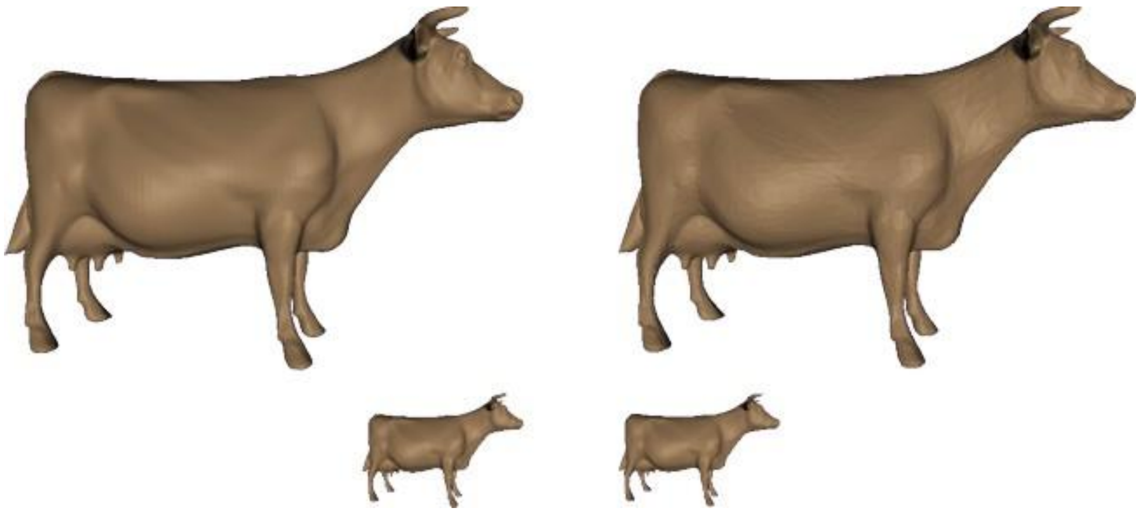
- Polygonal models used in practice are produced automatically by:
  - Rasterization of mathematically defined surfaces
  - 3D scanning of real objects
  - Other similar procedures
- The steady increase of computer power and the advances of 3D scanning lead to models that capture finest details, with larger number of vertices and faces
- *Digital Michelangelo* project, for example, used high-tech scanners to scan and reconstruct some Michelangelo sculptures
  - The triangle meshes produced contain several hundred million triangles
  - They occupy several gigabytes of data storage
  - This amount of information is difficult to process
  - This amount of detail is only useful in specific applications

# Polygonal Model Simplification (2)

- Computer graphics applications can benefit from multiple *resolutions* (*levels of detail* (LODs)) of the model that can be used in different viewpoint conditions:
  - When screen projection of a model is small, only a small amount of detail is discernible
- It would also be beneficial to vary the detail in different parts of the model:
  - Coplanar triangles could be merged into fewer and larger ones
  - Areas of the surface closer to the viewer would require more detail than those further away
- LODs and selective detail, explained above, are suitable for interactive applications displaying large graphics scenes

# Polygonal Model Simplification (3)

- Different LODs and sizes: (5000 vs. 1000 triangles)



- For the above reasons, several *model simplification* techniques have been developed
- They try to reduce number of faces of a polygonal model while retaining the appearance and structure of the original model
- Simplified models applications, usually employ several LODs of the original model and dynamically select the most suitable

# Polygonal Model Simplification (4)

- Model simplification techniques vary greatly in many respects:
  - Can be applied to different kinds of models
  - Take different paths for simplification of the models
  - Have different priorities and applications
- Simplification algorithms:
  - Deal most easily with closed manifold meshes
  - Handle, in most cases, the boundary of non closed models
  - Only few, are able to simplify non-manifold models

# Polygonal Model Simplification (5)

- Simplification methods can be classified in two main classes:
  - Those that produce *discrete* levels of detail of the initial model
  - Those that produce *continuous* levels of detail of the initial model
- Discrete levels of detail:
  - A target number of faces is prescribed
  - New model with the required number of faces is generated
  - If another level of detail is requested, the algorithm is executed again
- Continuous level of details:
  - A continuous sequence of increasingly simplified models is produced using local simplifications of the model
  - By recording the simplification steps, any intermediate level of detail may produced

# Polygonal Model Simplification (6)

- Continuous simplification algorithms are more interesting than discrete ones
- They are flexible and easily *reversible* → allow the application to move back and forth between level of details
- Support the *selective refinement*, enabling the dynamic adjustment of detail in different parts of the model
- Refine the mesh *smoothly* → minimizes visual artifacts due to switching resolution of the model in interactive applications

# Polygonal Model Simplification (7)

- Important issue for all simplification methods: *how to assess the quality of simplified model with respect to the original one*
- Most algorithms are guided by certain criteria in order to determine:
  - Where the "best" to put the new vertex is
  - Which edge should be removed first in order to minimize discrepancy
- These criteria also provide a global estimate of the quality of the algorithms → simplification algorithms can be compared

# Polygonal Model Simplification (8)

- Most widely used methods for this assessment is to measure some form of distance between the simplified and the original model:
  - *Hausdorff distance:* measures the maximum distance between any 2 points of 2 surfaces *M* and *M'*

$$d_\infty(M, M') = \max(\max_{\mathbf{v}\in M}\{d(\mathbf{v}, M')\}, \max_{\mathbf{v}'\in M'}\{d(\mathbf{v}', M)\}),$$

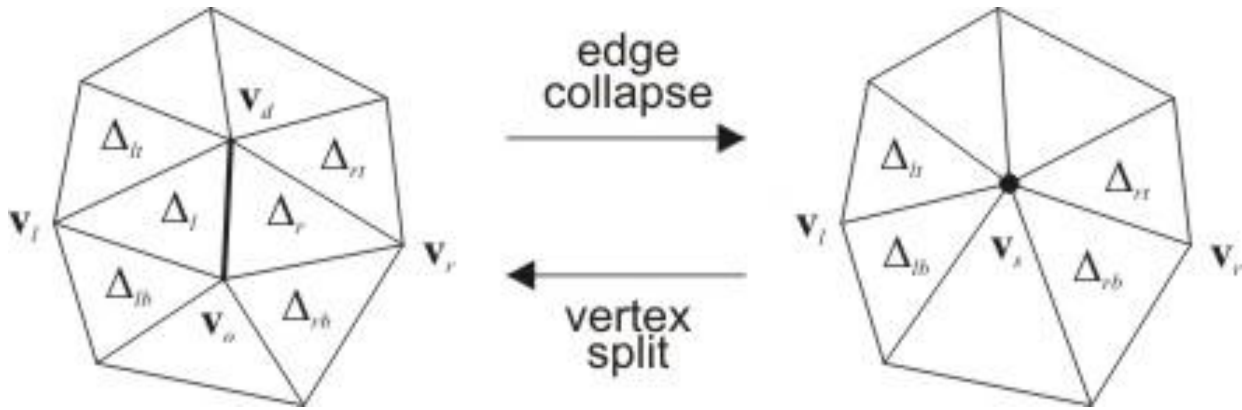
    where $d(\mathbf{v}, M) = \min_{\mathbf{w}\in M}\{|\mathbf{v}-\mathbf{w}|\}$ is the distance of a point **v** from a surface *M*, defined as the distance of **v** from the closest point **w** of the surface
  - *Mean square distance* of 2 surfaces:

$$d_2(M, M') = \frac{1}{s}\int_{\mathbf{v}\in M} d(\mathbf{v}, M') + \frac{1}{s'}\int_{\mathbf{v}'\in M'} d(\mathbf{v}', M),$$

  where *s* and *s'* are the areas of *M* and *M'* respectively
  - These formulae must be discretized in order to be computed on polygonal models → accomplished by sampling a number of points on both surfaces and using them for the computations

# Simplification using Iterative Edge Collapses

- Edge collapse:
  - Local operation on a triangle mesh
  - Removes an edge of the model and the 2 adjacent triangles by collapsing an edge to a single vertex




- Using edge collapses, it is easy to compute the distance between the simplified and the original mesh → only difference on the faces around the collapsed edge
- Variations of this method support non-manifold models
- We will concentrate on manifold models

# Simplification using Iterative Edge Collapses (2)

- The algorithm is summarized as follows:

1. For each edge of the model that can be collapsed, compute a collapse priority and sort the edges in a priority queue
2. While more candidate edges exist in the queue and the simplification target (maximum error, number of faces of the mesh) is not reached:
   a) Remove from the queue the edge collapse with highest priority
   b) Collapse this edge (mesh only changes locally around the edge)
   c) Re-compute the priorities of all edges affected by the collapse

- Two factors that affect the result of this method are:
  - The measure used to assess each edge collapse and assign its priority
  - The position of the new vertex for each edge collapse
- Different techniques have been proposed for the above 2 elements of the method

# Simplification using Iterative Edge Collapses (3)

- In some implementations, the position of the new vertex is fixed
- In other implementations, the above 2 factors are interrelated:
  - The position of the new vertex is computed as a result of an optimization procedure, minimizing the approximation error
  - The maximum error attained is used as the priority of the edge collapse

# Simplification using Iterative Edge Collapses (4)

- *Quadric error-metric* method:
  - minimizes the square distance of the new vertex from the faces around the collapsed edge
  - Let $\Delta$ be a triangular face of a model with plane equation:

$$ax + by + cz + d = 0$$

  - Squared distance of a point $\mathbf{x} = [x, y, z]^{\mathrm{T}}$ from the plane of $\Delta$ is:

$$Q_{\Delta}(\mathbf{x}) = \frac{(ax+by+cz+d)^2}{a^2+b^2+c^2} = \frac{(\vec{\mathbf{n}}^T\mathbf{x}+d)^2}{|\vec{\mathbf{n}}|^2} = (\hat{\mathbf{n}}^T\mathbf{x}+\hat{d})^2 = \mathbf{x}^T(\hat{\mathbf{n}}\hat{\mathbf{n}}^T)\mathbf{x} + 2\hat{d}\hat{\mathbf{n}}^T\mathbf{x} + \hat{d}^2,$$

where $\hat{\mathbf{n}} = \dfrac{\vec{\mathbf{n}}}{|\vec{\mathbf{n}}|}$ is the unit normal vector of $\Delta$ and $\hat{d} = \dfrac{d}{|\vec{\mathbf{n}}|}$

  - It can also be represented by the quadratic form:

$$Q_{\Delta} = (\mathbf{A}, \mathbf{b}, p) = (\hat{\mathbf{n}}\hat{\mathbf{n}}^T, \hat{d}\,\hat{\mathbf{n}}, \hat{d}^2),$$

so that:

$$Q_{\Delta}(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x} + 2\mathbf{b}^T\mathbf{x} + p$$

# Simplification using Iterative Edge Collapses (5)

- The sum of the squared distances of x from 2 triangles *Δ1* and *Δ2* can be computed by summing coordinate-wise the quadratic forms: $Q_{\Delta_1} = (\mathbf{A}_1, \mathbf{b}_1, p_1)$ and $Q_{\Delta_2} = (\mathbf{A}_2, \mathbf{b}_2, p_2)$ :

$$Q_{\Delta_1}(\mathbf{x}) + Q_{\Delta_2}(\mathbf{x}) = (Q_{\Delta_1} + Q_{\Delta_2})(\mathbf{x}) = \mathbf{x}^T(\mathbf{A}_1 + \mathbf{A}_2)\mathbf{x} + 2(\mathbf{b}_1 + \mathbf{b}_2)^T\mathbf{x} + (p1 + p2)$$

- The above is also a quadratic form
- This result generalizes naturally to any number of triangles
- The simplification algorithm assigns initially to each vertex **v** of the mesh, the form that expresses the sum of squared distances of a point from the faces around the vertex:

$$Q_{\mathbf{v}} = \sum_{\Delta \ arround \ \mathbf{x}} w_{\Delta} Q_{\Delta}$$

where $w_{\Delta}$ is a surface weight of the respective face → better scaling

# Simplification using Iterative Edge Collapses (6)

- Then, when an edge $e(\mathbf{v}_0, \mathbf{v}_d)$ is collapsed, the total squared distance of the resulting vertex $\mathbf{v}_s$ from all the faces around $\mathbf{v}_0$ and $\mathbf{v}_d$ is:

  $Q(\mathbf{v}_s) = Q_{\mathbf{v}_0}(\mathbf{v}_s) + Q_{\mathbf{v}_d}(\mathbf{v}_s)$ or $Q = Q_{\mathbf{v}_0} + Q_{\mathbf{v}_d}$

  which is the familiar form $Q = (\mathbf{A}, \mathbf{b}, p)$
- The position that minimizes $Q$ is the optimal for $\mathbf{v}_s$
- Minimum of $Q$ is attained at $\mathbf{v}_s = \mathbf{A}^{-1}\mathbf{b}$ and the minimum is :

  $Q(\mathbf{v}_s) = -\mathbf{b}^T \mathbf{A}^{-1}\mathbf{b} + p = \mathbf{b}^T \mathbf{v}_s + p$
- If $\mathbf{A}$ is a singular matrix → minimization is restricted along the edge $e(\mathbf{v}_0,\mathbf{v}_d)$
- If this fails, $\mathbf{v}_s$ is selected between $\mathbf{v}_0$ and $\mathbf{v}_d$ , depending on which vertex gives smaller value for $Q$

# Simplification using Iterative Edge Collapses (7)

- Simplification based on iterative edge collapses has all the properties of continuous level of detail methods:
  - It is easily reversible by performing *vertex split* in reverse order to the corresponding edge collapses → original positions must be kept with each edge collapse
  - By retaining some more information on the neighboring vertices and faces of each collapsed edge, it is possible to perform selective refinement and coarsening of the mesh on region of interest
  - Various error metrics and vertex-positioning strategies may be employed, so the method can be adapted to various interests and available resources

# Simplification using Iterative Edge Collapses (8)

- The simplification of large models is a lengthy operation
  - If an optimization procedure is used, it is even more costly → typically *performed offline*
  - *At run time*, generated levels of detail can be exploited *interactively* in real time for selectively refining the model

- Simplification based on edge collapses is becoming a standard feature in several graphics packages (e.g. DirectX)